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Full Length Research paper

A new performance index formulation aiming to attain fully stressed designs for topology optimization problems

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Evolutionary structural optimization (ESO) is a method combining topology and shape optimization. Although ESO has appeared on a simple foundation, many researchers up to now have made many contributions and performance based optimization, which incorporates performance indices to decide the best topology through all the designs provided by the optimization process, has been presented. Some criterions that reflect the structural behavior have been included in these procedures, however, all the former indices developed, take just one of these criterions into consideration such as stress, strain energy, displacement, etc. This paper presents a new performance index formulation, which is used to select the optimum design from the optimization process by considering the fully stressed design concept as the most affective criterion and deals with the previously mentioned problem by reviewing earlier studies. The efficiency and capability of the new index are demonstrated by several design examples.

Key words: Topology optimization, evolutionary method, performance index, fully stressed design.

INTRODUCTION

Many optimization methods have been developed for structural optimization up to now. Various size, shape and topology optimization methods aiming feasibility, less material usage and stiffness increment, have been taken into interest by researchers in an extensive field.

On the other hand, all the optimization methods face the same problem. Which one of the optimized designs is the best or how close is it to the optimum? Some mathematical methods are used occasionally to reply this question but no absolute answer is still possible. Consequently, all the environmental factors (that is, object function, subjects, and optimization parameters) cause very different designs.

The aim of the evolutionary structural optimization is to get the lightest design while satisfying the stiffness requirements and converging a fully stressed design. Because existing studies cannot perform absolute solutions about evaluating the performance of the optimum structures, a new performance index formulation is a necessity.

This paper presents a review of the performance index concept and a new method for assisting the determination of the best/optimum design throughout the optimazation process. However, it is possible to qualify this new formulation as an intuitive approach, but authors already think that the performance index concept should further be improved. Although the proposed procedure is experienced on ESO algorithm, it is possibly utilizable for many optimization methods by performing some modifications. Several design examples especially considering earlier examples which have been published in the literature, are provided to demonstrate and compare the effectiveness of the new methodology.

EVOLUTIONARY STRUCTURAL OPTIMIZATION

In the past two decades, significant progress has been made in the area of structural optimization, which aims at achieving the best structural performance by appropriate material distribution (Li et al., 1999a). Evolutionary structural optimization (ESO) presented by Xie and Steven (1993) and based on the simple idea that the

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optimal structure (maximum stiffness, minimum weight, that is) can be produced, by gradually removing the ineffectively used material from the design domain (Xie and Steven, 1997), is essentially based on two methods. Michell's truss theory, which is one of them, proposes optimal solutions with respect to the amount of material used and the constant stress state of all the members that constitute the structure. Moreover, the other basis, homogenization method that has been presented by Bendsøe and Kikuchi (1988), aims to get the optimum structural design by forming microscale voids throughout the optimization process.

The main algorithm of ESO is to carry out the finite element analysis and remove the elements iteratively, which are not effectively used. This idea has been derived from the idea of homogenization method, which aims to form voids at the understressed regions of the structure. In a manner of speaking, homogenization and ESO methods have a kind of partnership on the focus of this porosity idea.

A design domain covering the final design should be constituted before the optimization process and finite element analysis is carried out by applying the loads and boundary conditions. The inefficient material is removed from the structure by comparing the results of the analysis like stress, strain energy, displacement. However, it is an important point that ESO is applied so that the elements corresponding to the design domain are equally sized to get reliable optimization results. If this requirement is not met, the rejection and element removal criterions should be altered in a way of considering the varying sizes of the elements to deal with this situation (Tanskanen, 2002).

The stress level of each element is determined by comparing the von Mises stress of the elements σ_e^{vm} to the maximum von Mises stress of the whole structure $\sigma_{\max}^{vm}$. At the end of each finite element analysis, all the elements that satisfy the following condition are deleted from the model:

$$\sigma_e^{vm} < RR_i \cdot \sigma_{\max}^{vm}, \quad (1)$$

where RR_i is the current 'rejection ratio' (RR).

The cycle of finite element analysis and element removal is repeated using the same value of RR_i until a 'steady state' is reached, which means that there are no more elements being deleted at the current iteration. At this stage, an 'evolutionary rate' (ER) is introduced and added to the rejection ratio:

$$RR_{i+1} = RR_i + ER \qquad i = 0,1,2,3,\ldots \quad (2)$$

With this increased rejection ratio, the cycle of finite element analysis and element removal takes place again until a new steady state is reached (Xie and Steven, 1997).

In finite element analysis, the element absence or presence can be simply represented by a property of type 0 or 1 (Li et al., 1999b). In addition to this "hard-kill" method, another way of 'removing' an element is to reduce its elasticity modulus, density or dimensions such as element thickness to a very small value and should be called as "soft-kill" method. For instance, Hinton and Sienz (1995) reduce the elasticity modulus of the elements to be removed by a factor of 10^{-5} or 10^{-6} etc. However, the most practical way is to modify the element properties. In fact, any parameter that affects the structural behavior of the design can potentially be included for changing the contribution degree of the elements to the whole design. In consequence of removing the elements by assigning the material property number to 0, some elements having insufficient connection to other elements by only one of its nodes may cause singularity of the stiffness matrix in subsequent analyzes (Özkal, 2006). To avoid an extra operation of checking the connectivity of elements, changing the elasticity modulus has been taken as the element removing method for this study.

ESO works by attempting to imitate the growth of biological structures in nature. It was observed that naturally occurring species tend to achieve shapes that are close to 'fully stressed' configurations as this leads to the optimum material utilization (Das et al., 2005). Removing the elements that are not effectively used and getting the minimum stress that increased up to higher values, constitute relatively fully stressed designs and additionally, keeping the maximum stress values close to the initial values which expands the applicability of the optimum designs. In other words, optimized designs that have the least weight (volume), supports the loads by the most effective topology.

ESO can be used with various design objective functions and constraints such as stress, stiffness, displacement, frequency, buckling load, moment of inertia and thermal parameters (Das et al., 2005). Besides, during the evolutionary process, it is not necessary to generate a new mesh for each of the iterations. That is one of the major superiorities of the evolutionary optimazation related to the computation time (Xie and Steven, 1993).

ESO method has an assistant characteristic in any industrial field and the optimized designs can be used by making some changes due to the necessities. Additionally, the manufacturers may assume unsmooth boundaries of the optimum designs, achieved by ESO as a problem but some researchers in the literature have presented solutions for this difficulty.

DEVELOPMENT HISTORY OF THE PERFORMANCE INDEX CONCEPT

Rise of the optimization concept's popularity over the engineering science comes along with various arguments. An important one of these arguments is about the decision on the optimality of the designs formed by optimization procedures. Some studies on this matter have been done for both ESO and the other optimization methods presented in the literature.

While the elements are removed after each finite element analysis throughout the optimization process, the weight of the structure is decreased, maximum-minimum stress values become closer, and the performance of the new design is increased. In a manner of speaking, the efficiency of the material usage for the new design increases iteratively as expected. Because all the newer designs have better topologies than the previous ones, all of them can be selected as an optimum result of the procedure. Besides, some specific criterions should be established. Especially the fully stressed state should be examined initially and final selection is made by considering weight (volume), stiffness, wholeness of the structural parts and also the applicability.

A performance index (PI), which select the best of the optimum designs generated by ESO, has been developed firstly by Querin (1997). Subsequent studies have presented new performance index formulations but all of them consider the displacement value of the loaded point or the maximum stress throughout the whole structure as relative stiffness next to the weight of the structure. However, the optimality/goodness of the design is not connected to these parameters separately; the ones which contribute the structural performance should be taken into consideration altogether in someway and a generalized determination method is needed.

According to Querin's formulation, the best-optimized structure can be found by selecting the design that has the lowest value of PI:

$$PI = \frac{\sum_{e=1}^{N} \sigma_e V_e}{FL} \tag{3}$$

where F is the resultant applied load to the structure, L is a nominal length for the structure, is the von Mises stress in the eth element, Ve is the volume of the eth element and N is the number of elements in the structure. However, this formula is not linked with any optimization constraint (Liang et al., 1999) and not sufficient in adaptation to many design optimization problems in opposition to the assertion.

Even it was not named as performance index; Zhao et al. (1998) have presented a new formulation. The average structural stiffness per unit volume is used to express how efficiently the structural material is used and to assess the quality of the solutions from the evolutionary optimization method. Since the overall stiffness of a structure is inversely proportional to the total work done by all the external loads in the structure, Zhao et al. (1998) define the normalized material efficiency indicator of a structure as:

$$\psi = \frac{W_E^0 \, V^0}{W_E^s \, V^s} \tag{4}$$

where W_E^0 and W_E^s are the total work done by all the external also V^0 and V^s are the volumes in the initial design domain and the current structure, respectively.

Liang et al. (1999) have presented a new index for the optimization process of continuous structures under stress constraints. The derivation of the performance index is firstly formulated based on the scaled design concept, and it is used to identify the optimal topologies of various continuum structures with stress and height constraints and to compare the efficiency of structural topologies obtained by different methods. By assuming the maximum stress value in the structure as the most active constraint, the stress based performance index at the *i*th iteration is proposed as following so that the requirement on the strength limit state needs to be satisfied;

$$PI_s = \frac{\sigma_{0,\max}^{\mathrm{vM}} \, V_0}{\sigma_{i,\max}^{\mathrm{vM}} \, V_i} \tag{5}$$

$\sigma_{0,\max}^{\mathrm{vM}}$ and $\sigma_{i,\max}^{\mathrm{vM}}$ are the maximum von Mises stress values, while V_0 and V_i are the volumes of the initial and current designs at the *i*th iteration, respectively.

With respect to the above formula, the performance index is equal to unity for the initial design. Minimizing the weight of a structure with stress constraint can be achieved by maximizing the performance index in the optimization process. Although researchers have stated that the performance index presented can indicate the uniformity of stress within the optimal topology of continuum structures, it will be shown in the next section that solutions, which are more reliable, are possible in the determination of best designs.

Liang et al. (2000a) have developed another index as an indicator of material efficiency and a termination criterion of the process so that the global optimum design can be determined by assuming the displacement value of the load point as the most active constraint over the structural behavior. Displacement based performance index is defined as:

$$PI_{ds} = \frac{u_{0j} \, W_0}{u_{ij} \, W_i} \tag{6}$$

where u_{0j} and u_{ij} are the absolute values of the *j*th constrained displacement, which is basically the load point, while W_0 and W_i are the actual weights of the initial and current designs at the ith iteration, respectively.

The performance index of the initial design is equal to unity, same as the index in the Formula 5. The performance of a structural topology is improved when inefficient materials are removed from the design domain. This is the most frequently used index for the ESO method as seen in many of the papers in the literature. In case the Formulas 4, 5 and 6 are examined, total work, maximum stress and displacement values increase but weight/volume of the design decreases faster in comparison with the design of former iterations.

Furthermore, an altered index formulation for the plate bending problems aiming optimal thickness distribution has been presented by Liang et al. (2001) based on scaled design approach, too. Because the stiffness matrix of a plate in bending is not a linear function of the design variable such as the thickness of the plate, the performance index that is evaluated by the constrained displacements and the volumes at each iteration is expressed as:

$$PI_{dp} = \left(\frac{|u_{0j}|}{|u_{ij}|} \right)^{1/3} \frac{V_0}{V_i} \tag{7}$$

A similar approach has also been presented by Liang et al. (2000b) on the research of linear elastic continuum structures. The optimization algorithm, which aims a definite stiffness level for the final design, is based on removing the elements that have the lowest strain energies. Hence, the energy based performance index is stated as:

$$PI_{es} = \frac{C_0 W_0}{C_i W_i} \tag{8}$$

where C_0 and C_i are the mean compliance values, and W_0 and W_i are the weight values of the initial and current designs at the ith iteration, respectively. Moreover, Liang and Steven (2002) define the performance index for plates in bending as follows, owing to the nonlinearity of the stiffness matrix:

$$PI_{ep} = \left(\frac{C_0}{C_i} \right)^{1/3} \frac{W_0}{W_i} \tag{9}$$

Principal stresses have been taken into consideration to develop a performance index by Guan et al. (2001) in order to design tension- and compression-dominant structures. The algorithm works by removing the materials that have the lowest principal stress values ($|\sigma_{11}^{e}|$ or $|\sigma_{22}^{e}|$ according to the type of design). The performance indices, which measure the efficiency of the structural topologies and indicate the global optimum designs at its highest values, are based on volume and principal stresses and defined as follows:

For tension-dominant designs:

$$PI_t = \frac{|\sigma_{22,\max}|_0 \, V_0}{|\sigma_{22,\max}|_i \, V_i} \tag{10}$$

and for compression-dominant designs:

$$PI_c = \frac{|\sigma_{11,\max}|_0 \, V_0}{|\sigma_{11,\max}|_i \, V_i} \tag{11}$$

Briefly, each one of the performance indices above is an innovation on its own and an aid tool for the selection of the best designs among the optimization history. Nevertheless, the common deficiency is the consideration of only one criterion for each index. Besides, the Formula 6 (PI_{ds}), which considers the displacement value of the load point and the Formula 8 (PI_{es}), which considers the mean compliance, both exhibit the same results with the Formula 4 that has been (ψ) developed by Zhao et al. (1998). Because the formula of Liang et al. (2000a) is based on displacement of the load point and the formula of Liang et al. (2000b) is based on mean compliance in response to the formula of Zhao et al. (1998), which is based on the total work done by external forces. Numerical results of all these formulas overlap each other as can be seen in the design examples section.

A NEW PERFORMANCE INDEX FOR FULLY STRESSED DESIGN

It is a very important stage to determine the optimum design among the results of the optimization history. In case of a change for one of the parameters for the optimization process, it is possible to get various designs. In addition, even if there is no change for the parameters, designs for all the iterations are so distinctive in comparison to each other. Hence, it is a prior necessity to define exactly what “optimum” means.

Main goal of ESO is to reduce the weight of the structure but stiffness is also one of the most important constraints. It is expected that the structure preserves its stiffness, namely, the displacement of the nodes should not have excessive values, while the weight (volume) of the structure goes down. Because the loading and the boundary conditions are constant, the optimization process will provide no benefit if the maximum displacement of the nodes exceeds the limit.

It is another important goal for the optimization process, to constitute a fully stressed design that means the whole material or parts of the structure are under stress equally or closely. In a different manner, it is desired that the material should be used in the most effective way. Even the fully stressed design concept is more in use for truss systems; it is appreciated as a significant factor for ESO because of its capability of producing truss-like designs.

The early versions, especially the stress-based version of ESO use the RR parameter as a criterion for termination of process and determination of the optimum design by proportioning the maximum stress to the minimum throughout the whole structure.

Another factor for the assessment of the optimum design is manufacturability. This question should be answered: “Is the achieved design convenient for the application in needed scopes or not?” Nevertheless, manufacturers will make the best judgment, and the decision of the designer will be just a proposing comment, but nothing else based on their experience and engineering intuition.

In order to summarize and show all the cruxes about the necessity of a performance index, a new formula should be built up by considering weight (volume), stiffness and the stress levels within the structure. As mentioned above, all the approaches until today have a necessity about the determination of the best design and miss some points for this objective. Even so, these methodologies have given rise to the performance index concept and facilitated the latter studies.

First of all, the objective and the constraints of the topology optimization problem should be defined for all the performance criterions:

$$\text{minimize} \quad W = \sum_{e=1}^{N} w_e \tag{12}$$

$$\text{subject to} \quad u_{\max} \le u^*_{\max} \Rightarrow \frac{u_{\max}}{u^*_{\max}} \le 1 \tag{13a}$$

$$\sigma_{\max} \le \sigma^*_{\max} \quad \Rightarrow \quad \frac{\sigma_{\max}}{\sigma^*_{\max}} \le 1 \tag{13b}$$

$$\sigma_{avg} \ge \sigma^*_{avg} \quad \Rightarrow \quad \frac{\sigma^*_{avg}}{\sigma_{avg}} \le 1 \tag{13c}$$

where *W* is the total weight of a structure, w_e is the weight of the *e*th element and *n* is the total number of elements in the design. In addition, $u_{\max}$ is the maximum displacement, $\sigma_{\max}$ is the maximum von Mises stress and σ_{avg} is the average of the von Mises stresses of elements throughout the whole structure while $u^*_{\max}$, $\sigma^*_{\max}$ and σ^*_{avg} are the prescribed limits of the aforesaid constraints.

By using the displacement constraint as the first scaling criterion, the scaled weight of the initial design domain can be expressed similarly by the prior approaches:

$$W_0^{s,I} = \left(\frac{u_{0,\max}}{u^*_{\max}}\right) W_0 \tag{14}$$

W_0 is the actual weight of the initial design domain, and $u_{0,\max}$ is the maximum displacement value, which is the most critical in the initial design under real loads.

Previous studies about performance index consider the displacement of the load point for this formula. However, it is not possible to detect the maximum deflection point of structure for all the iterations. Because the structural topology varies a lot in comparison to the initial design, while the optimization process goes on. Consequently, it is more appropriate to check the analysis results after each iteration and determine the value of the maximum displacement among the whole structure.

When the maximum stress criterion is used to extend the above expression, subsequent scaled weight of the initial design domain is defined by the addition of maximum von Mises stress ($\sigma_{0,\max}$) as following:

$$W_0^{s,II} = \left(\frac{u_{0,\max}}{u^*_{\max}}\right)\left(\frac{\sigma_{0,\max}}{\sigma^*_{\max}}\right) W_0 \tag{15}$$

Eventually, the most effective part of the expression, which is dependent upon σ_{avg} , is defined:

$$W_0^{s} = W_0^{s,III} = \left(\frac{u_{0,\max}}{u^*_{\max}}\right)\left(\frac{\sigma_{0,\max}}{\sigma^*_{\max}}\right)\left(\frac{\sigma^*_{avg}}{\sigma_{0,avg}}\right) W_0 \tag{16}$$

where $\sigma_{0,avg}$ is the average von Mises stress value of all the elements in the structure. The last part of the expression, which is mentioned above, is especially important for the fully stressed design concept. It is expected that the average stress of the structure gets closer to the maximum stress during the optimization process to provide a more uniform stress distribution. In determining how close the ratio of $\sigma_{\max}$ to σ_{avg} is to unity, material of the structure is used more effectively. This ratio can be assumed like a ratio of the stress density.

The scaled weight of the current design at the *i*th

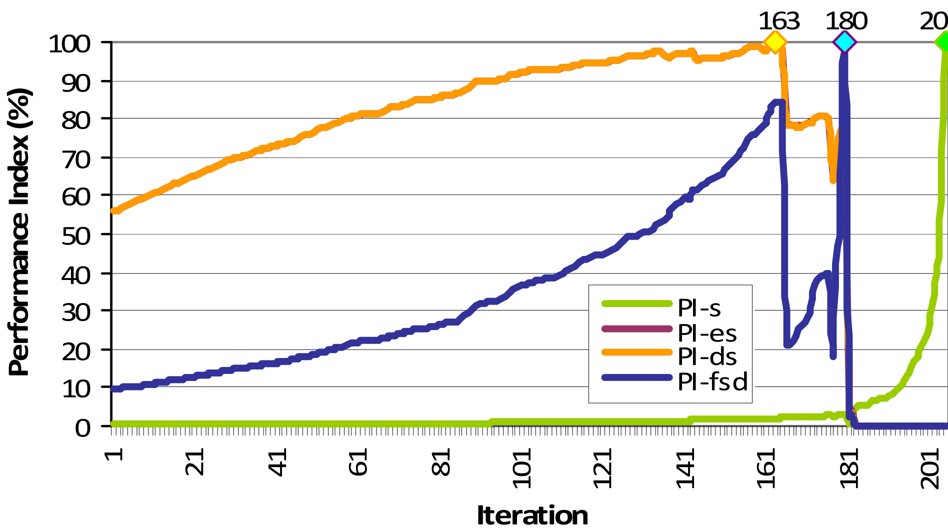


Figure 1. Performance index histories for Example 1.

iteration can be denoted similarly as:

$$W_i^s = \left(\frac{u_{i,\max}}{u_{\max}^*}\right)\left(\frac{\sigma_{i,\max}}{\sigma_{\max}^*}\right)\left(\frac{\sigma_{avg}^*}{\sigma_{i,avg}}\right)W_i \tag{17}$$

where W_i is the actual weight, $u_{i,\max}$ is the maximum displacement, $\sigma_{i,\max}$ is the maximum von Mises stress of an element and $\sigma_{i,avg}$ is the average von Mises stresses of all elements in the current design at the *i*th iteration under the applied loads.

Finally, the performance index, which is based on displacement constraints and the uniform distribution of the interior stress values, is formulated by the proportion of the scaled weights of the initial to the current design:

$$PI_{fsd} = \frac{W_0^s}{W_i^s} = \frac{\left(\sigma_{0,\max}^{\mathrm{vM}} / \sigma_{0,avg}^{\mathrm{vM}}\right) u_{0,\max}\, W_0}{\left(\sigma_{i,\max}^{\mathrm{vM}} / \sigma_{i,avg}^{\mathrm{vM}}\right) u_{i,\max}\, W_i} \tag{18}$$

The performance index developed in this study is named as PI_{fsd} to demonstrate the effect of the fully stressed design concept. It evaluates the structural performance of the design based on the stiffness and stress uniformity considerations and is used for the determination of the optimum design by selecting the one which has the greatest PI_{fsd} value. It has been examined on various optimization problems to see if successful results are acquired or not and compared to other indices in order to exhibit the distinction of the new formula. Briefly, the results are quite advantageous.

DESIGN EXAMPLES

Design examples in this study are selected from the recent studies to demonstrate the advantage of PI_{fsd} more clearly. All the examples presented in this section have been presented previously and some other researchers tested the same examples to prove their improvement approaches or show the innovations from ESO method. These are also important examples to demonstrate the ability of ESO to produce truss-like designs.

Example 1

Concentrated load of F= 100 kN is applied at the middle of the bottom of the 10 × 5 m beam and corners are assumed as fixed. Thickness is 0.1 m, Young modulus is 100 GPa and Poisson's ratio is 0.3. Static analysis is carried out by using a mesh of 1250 (50 × 25) square four-node plane stressed elements. To minimize the weight of the structure and make the structure almost fully stressed, stress based criterion has been used for the evolutionary optimization. To avoid an extra operation of checking the connectivity of elements, changing the elasticity modulus (soft-kill method) has been preferred for removing element in this study. In other words, although element existence can be chosen, elasticity modulus of each element has been preferred as the design variable, and they have been multiplied by a factor of 10^{-6} for removal.

All the elements are removed after 205 iterations and the performance index histories for the beam are presented in Figure 1 to compare the results of different indices. PI values are given in percentage proportion to the maximum value of each index to show all the index

Table 1. Performance index values at specific iterations for Example 1.

Iteration	PI_s	PI_{es}	PI_{ds}	PI_{fsd}	V_{max} (MPa)	V_{avg} (MPa)	V_{min} (MPa)	u_{max} (mm)	W (%)
0	1	1	1	1	3.35	0.26	0.001	0.07	100
163	5.01	1.79	1.79	8.93	4.02	1.57	0.81	0.24	17
180	7.47	1.41	1.41	10.62	5.00	2.97	1.39	0.56	9
205	242.50	0.001	0.001	0.004	-	-	-	-	-

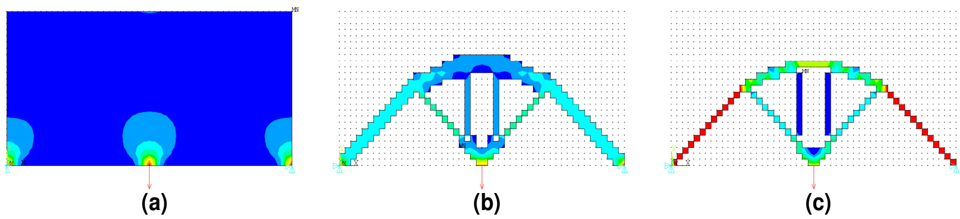


Figure 2. Initial and optimum designs for Example 1. (a) Initial design; (b) Design at 163rd iteration (c) Design at 180th iteration

histories and the peak points in a figure. Furthermore, Table 1 shows the values of these indices as well as stress, displacement and weight values at specific iterations. However, the results of the performance index that is based on principal stresses are not presented here because algorithm contains distinct objectives and constraints, and the comparison of this index with the others will not be so meaningful.

If Figure 1 and Table 1 are examined, the results of PI_{es} and PI_{ds} overlap each other as an expected occurrence and the peak values concur at the 163rd iteration. In addition, PI_s has an increasing tendency and the peak value occurs at the 205th iteration, however, it is the last design in the optimization process and the design that has only four elements cannot be accepted as a manufacturable design. Thus, there is no need to check up the last iteration and represent the numerical results. Nevertheless, PI_{fsd} represents the best design at 180th iteration. It is worth to say that the same design has also been presented as the optimum design by Xie and Steven (1993) but by considering the *RR* values as the termination criterion. It is obvious that the design, which is offered by PI_{fsd}, is more appropriate according to the fully stresses design aspect and numerical results. Initial and intermediate designs by the stress distribution illustration can be found in Figure 2.

Example 2

The same beam with the same loading and analysis conditions is used for the second example but only the right support is replaced from fixed to rolling. The optimization process has come an end after 156 iterations. Comparison of performance indices, index values and numerical results and optimum designs can be found in Figure 3, Table 2 and Figure 4, respectively.

PI_{fsd} points the best design at 150th iteration, while PI_{ds} and PI_{es} point the one at 132nd iteration. The offer of PI_s is again the last iteration, which does not have a logical topology. The similarity of the PI_{fsd} design to the truss structures and the uniform distribution of the interior stresses are remarkable like the previous example.

Example 3

A concentrated load of F = 100 kN is applied at the middle of the right of the 10 × 24 m cantilever beam that has the same mechanical properties and the left side of the beam is completely fixed. Static analysis is carried out by using a mesh of 1500 (25 × 60) square four-node plane stressed elements. Comparison of performance indices, index values and numerical results and optimum designs can be found in Figure 5, Table 3 and Figure 6, respectively.

All the performance indices agrees at the same design for this problem. $PI_{fsd,}$ PI_{ds} and PI_{es} point the best design at 251st iteration and the offer of PI_s is the last iteration, which does not have a logical topology as expected. In addition, 244th design is shown in Figure 6 as suggested because there is a considerable drop for the indices after this iteration.

RESULTS AND DISCUSSION

It is a common problem for all the optimization methods to decide how good or how applicable are the designs obtained in a process. Because of the difficulties that

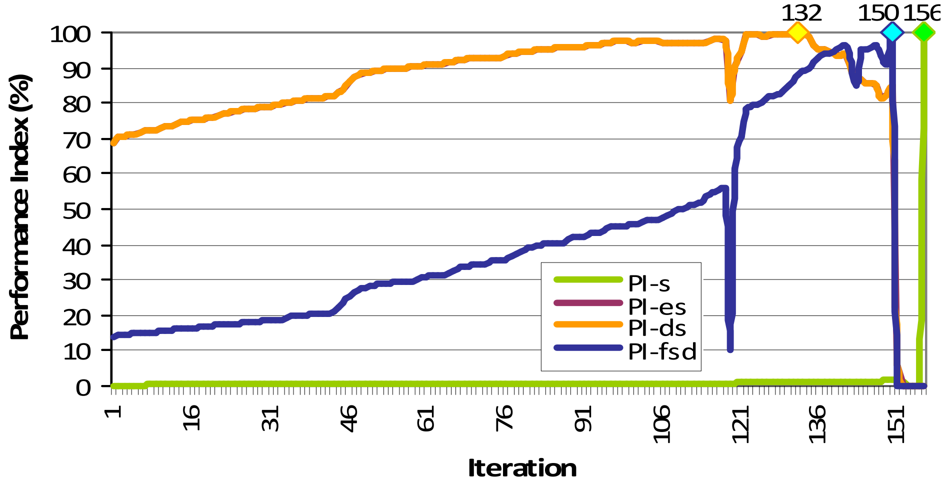


Figure 3. Performance index histories for Example 2.

Table 2. Performance index values at specific iterations for Example 2.

Iteration	PI_s	PI_{es}	PI_{ds}	PI_{fsd}	V_{max} (MPa)	V_{avg} (MPa)	V_{min} (MPa)	u_{max} (mm)	W (%)
0	1	1	1	1	3.51	0.32	0.002	0.10	100
132	4.12	1.45	1.45	6.35	3.53	1.38	0.72	0.28	24
150	5.89	1.23	1.23	7.24	3.51	1.97	1.04	0.50	17
156	349.06	0.001	0.001	0.003	-	-	-	-	-

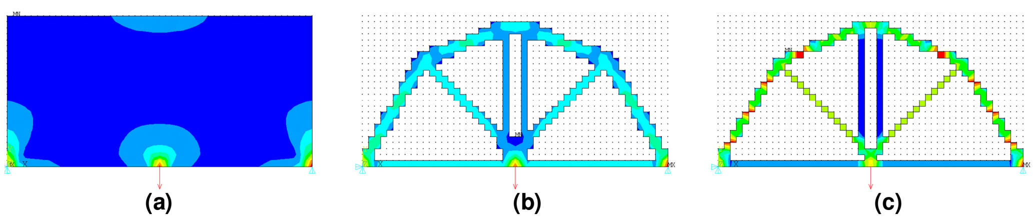


Figure 4. Initial and optimum designs for Example 2. (a) Initial design; (b) Design at 132nd iteration; (c) Design at 150th iteration.

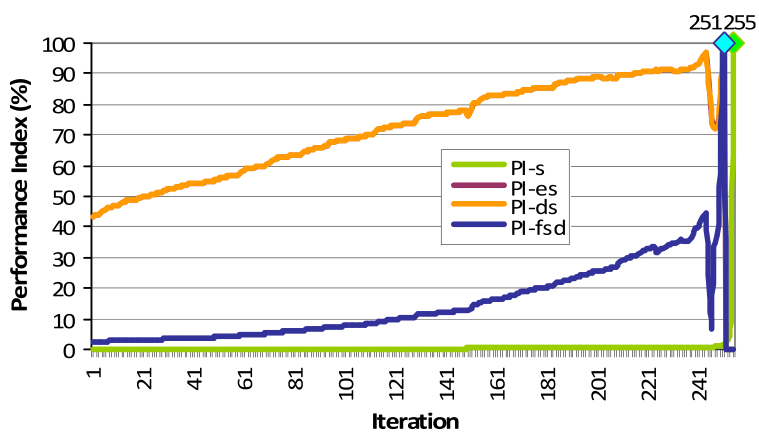
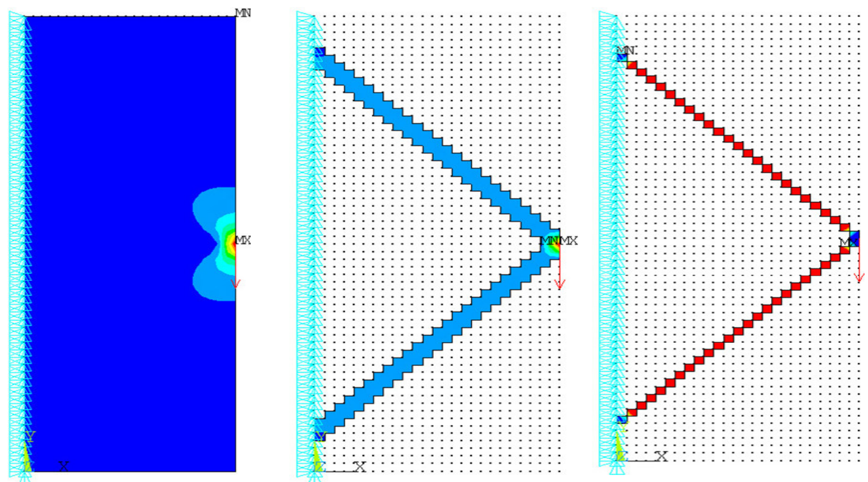


Figure 5. Performance index histories for Example 3.

Table 3. Performance index values at specific iterations for Example 3.

Iteration	PI_s	PI_{es}	PI_{ds}	PI_{fsd}	V_{max} (MPa)	V_{avg} (MPa)	V_{min} (MPa)	u_{max} (mm)	W (%)
0	1	1	1	1	1.64	0.09	0.001	0.04	100
244	9.08	2.24	2.24	18.91	1.83	0.84	0.67	0.17	10
251	19.64	2.32	2.32	42.26	2.50	2.49	2.45	0.50	3.3
255	1056.06	0.001	0.001	0.007	-	-	-	-	-



(a) (b) (c)

Figure 6. Initial and optimum designs for Example 3. (a) Initial Design; (b) Design at 244th iteration; (c) Design at 251st iteration.

recent approaches have with the determination of the best design in an optimization process, development of a new performance index was a necessity. Additionally, it is expected that this index work as a termination criterion for the process. Because large computation time appears as a big disadvantage, especially for the design domains, which have a great number of finite elements. In case there is no termination criterion to be used, the optimization process should wait till the end and this is not an expedient characteristic for any optimization method.

New performance index, which is named as PI_{fsd}, has been tested on various design problems such as the above examples and the results are very convincing. All the important considerations (fully stressed design, weight reduction, stiffness, entirety of the topology, similarity to the truss structures and the benefit in the computation time aspect) for any optimization method are obtained with the aid of PI_{fsd}.

Nevertheless, the performance index concept is receptive and should be further developed according to the additional necessities. More factors may be included in the index formulation. Additionally, proportion of the recent factors may be altered by considering specific design or loading/boundary conditions in future studies. Also as previously discussed, manufacturability of the designs should be concretized and inserted in the formulation in order to select the absolute optimum design in an exact way.

REFERENCES

Bendsøe MP, Kikuchi N (1988). Generating optimal topologies in structural design using a homogenization method. Comput. Meth. Appl. Mech. Eng. 71(2): 197-224.

Das R, Jones R, Xie YM (2005). Design of structures for optimal static strength using ESO. Eng. Fail. Anal. 12(1): 61-80.

Guan H, Chen YJ, Loo YC (2001). Topology optimization of bridge type structures with stress and displacement constraints. Int. J. Comput. Eng. Sci. 2(2):199-221.

Hinton E, Sienz J (1995). Fully stressed topological design of structures using an evolutionary procedure. Eng. Comp. (Swansea, Wales). 12(3): 229-244.

Li Q, Steven GP, Querin OM, Xie YM (1999b). Evolutionary shape optimization for stress minimization. Mech. Res. Commun. 26(6): 657-664.

Li Q, Steven GP, Xie YM (1999a). Displacement minimization of thermoelastic structures by evolutionary thickness design. Comp. Meth. Appl. Mech. Eng. 179(3-4): 361-378.

Liang QQ, Steven GP (2002). A performance-based optimization method for topology design of continuum structures with mean compliance constraints. Comput. Meth. Appl. Mech. Eng. 191(13-14):1471-1489.

Liang QQ, Xie YM, Steven GP (1999). Optimal selection of topologies for the minimum-weight design of continuum structures with stress constraints. In: Proc. Inst. Mech. Eng. Part. C. J. Mech. Eng. Sci. 213(8): 755-762.

Liang QQ, Xie YM, Steven GP (2000a). Optimal topology selection of continuum structures with displacement constraints. Comput. Struct. 77(6): 635-644.

Liang QQ, Xie YM, Steven GP (2000b). Optimal topology design of bracing systems for multistory steel frames. J. Struct. Eng. 126(7): 823-829.

Liang QQ, Xie YM, Steven GP (2001). A performance index for topology and shape optimization of plate bending problems with displacement constraints. Struct. Multi. Optim. 21(5): 393-399.

Özkal FM (2006). Optimum design of beams by evolutionary structural optimization. MSc dissertation, Department of Civil Engineering, Atatürk University, Erzurum, Turkey.

Querin OM (1997). Evolutionary structural optimization: Stress based formulation and implementation. PhD dissertation, Department of Aeronautical Engineering, University of Sydney, Sydney, Australia.

Tanskanen P (2002). The evolutionary structural optimization method: Theoretical aspects. Comput. Meth. Appl. Mech. Eng. 191(47-48): 5485-5498.

Xie YM, Steven GP (1993). A simple evolutionary procedure for structural optimization. Comput. Struct. 49(5): 885-896.

Xie YM, Steven GP (1997). Evolutionary Structural Optimization. Springer-Verlag, Berlin.

Zhao C, Hornby P, Steven GP, Xie YM (1998). A generalized evolutionary method for numerical topology optimization of structures under static loading conditions. Struct. Optim. 15(3-4): 251-260.