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Middle school mathematics teachers' pedagogical content knowledge regarding teaching strategies on quadrilaterals

Elif Nur Akkaş and Elif Türnüklü

Dokuz Eylul University, Izmir, Turkey.

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Pedagogical content knowledge is consisted of two components: student knowledge and teaching strategies. Teaching strategies was defined under two sub-headings as strategies for specific topics and specific strategies for any topic. The purpose of this study was to examine the method with which quadrilaterals were taught by mathematics teachers with regard to the teaching strategies component of pedagogical content knowledge. 30 middle school mathematics teachers working at 12 different schools in Turkey participated in this study. Interview method was used for data acquisition. The interview was intended to put forth the strategies that the teachers used for defining, classification and visualization which were included in the strategies of the framework used for the study. Content analysis was used to analyze the data acquired in this study. Study results showed that the definition strategy was used together with the visualization strategy. It was also determined that informal definitions were used in addition to formal definitions and that in general a personal definition was given by listing the various properties of quadrilaterals. The teachers who participated in this study generally used the partial classification of quadrilaterals and the transformation classification of quadrilaterals.

Key words: Pedagogical content knowledge, teaching strategies, quadrilaterals.

INTRODUCTION

Teacher education research has been suggested as a new field of study by Shulman (1986) and has created a framework to determine what teachers need and develop effective teaching according to some factors. In this framework, Shulman (1987) has examined pedagogical content knowledge as consisting of two components: student knowledge and teaching strategies. When components of the knowledge to understand students in various different pedagogical content knowledge models are examined (Grossman, 1990; Fennema and Franke, 1992; Schoenfeld, 1998; Magnusson et al., 1999; An et al., 2004; Ball et al., 2008; Park and Oliver, 2008; Kovarik, 2008) it is observed that student knowledge is examined in six different sub-components such as revealing the current knowledge of students, associating preliminary knowledge with new knowledge, valuing

*Corresponding author. E-mail: elifnurakdogan@hotmail.com

student questions and thoughts, taking into consideration the individual differences, predicting student thought and determining-knowing the misconceptions of students. It is also observed that the teaching strategies component in these models is examined under six different sub-components such as strategies for specific topics, specific strategies for any topics, demonstrations, (examples, real world problems, problems), analogies, illustrations, samplings, explanations.

Various researches have carried out on the pedagogical content knowledge components which was put forth by Shulman (Park and Oliver 2008, different components have been listed in detail by different researchers). In this examination, Park and Oliver (2008) have determined that many of these researchers generally use two components determined by Shulman and that in addition to these components they also create new components called knowledge of assessment of subject matter along with knowledge of curriculum.

When studies carried out on mathematics teachers are examined in terms of pedagogical content knowledge, it is seen that there are other studies that examine different topics in mathematics in terms of pedagogical content knowledge (Hacıomeroglu, 2009; Basturk, 2009; Yesildere and Akkoc, 2010; Bukova-Guzel, 2010). These studies have also put forth certain clues as to how education for this topic should be.

The determination of strategies for specific topics in geometry are important for effective teaching. When teaching the topics, the awareness of the teachers of these strategies, the usage of them appropriately and the pedagogical content knowledge are effective in terms of teaching strategies component. When the studies carried out are examined (Grossman, 1990; Fennema and Franke, 1992; Schoenfeld, 1998; Magnusson et al., 1999; An et al., 2004; Ball et al., 2008; Park and Oliver, 2008; Kovarik, 2008; Hacıomeroglu, 2009; Basturk, 2009; Yesildere and Akkoc, 2010; Bukova-Guzel, 2010) it is seen that the number of studies examining the subject of quadrilaterals in terms of pedagogical content knowledge is fairly low. The aim of this study is to examine middle school mathematics teachers' pedagogical content knowledge regarding teaching strategies on quadrilaterals.

Theoretical Framework

In this study, the teaching strategies sub-component of pedagogical content knowledge was used. Magnusson et al. (1999) have presented teaching strategies defined for use in science and mathematics education under two sub-headings as strategies for specific topics and specific strategies for any topics. Specific strategies for any topics are defined as strategies containing the various stages used in the teaching of a specific field whereas strategies for specific topics are defined as examples, models and activities used in teaching of a specific topic or concept.

The focus of this study was the teaching strategies sub-component of pedagogical content knowledge developed by Magnusson et al. (1999). Quadrilaterals have been selected in this study in the context of teaching strategies.

There are various studies on students' perceptions and concept images on geometry. When studies carried out on the learning of students for quadrilaterals are examined, it has been observed that specifically two factors played a role in understanding, perception and comprehension. These being the definition of the concept image and figural concept. Concept image put forth by Tall and Vinner (1981) is not only limited to the concept definition and they have defined concept image as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p. 152).

Individuals may perceive the same concept in different ways due to their individual epistemological and psychological attributes. Definition of the concept is defined as the whole set of words used to distinguish one concept from the others, whereas concept image is defined as what is conjured up in the mind for that concept either consciously or unconsciously. Concept image includes partially correct definitions and misconceptions.

Hershkowitz (1989, 1990), Tall and Vinner (1981) have interpreted their definition of concept image along with the definition of concept in critical attributes and non-critical attributes putting forth that non-critical attributes belong to the concept image. Geometrical shapes have certain visual structures in addition to certain formal definitions. Another important factor that should be considered when giving examples of geometrical shapes is that there may be one or more prototypes. Herskowitz (1989) has explained this by stating that all samples have common specific visual features thereby prototypes. Tsamir et al. (2008) have accepted prototype samples as the heuristic representative of the concept. In this sense, the prototype factors comprise the key factor. Each concept may have more than one prototype sample. These prototype samples are those that represent some of the features included in the long list of features for the concept. These prototype figures always have an effect on the concept image (Fischbein, 1993; Hershkowitz, 1990).

Many geometrical figures contain familial relations. Hence, the apprehension of these familial relations is important in the mathematics curriculums for the teaching of geometrical concepts. The classification of quadrilaterals is as important as definition and visual properties. The classification of quadrilaterals seems to be important to create relations between quadrilaterals and thus for the solution of geometrical problems along with proof studies. That is why it has been the focus point of many studies. De Villiers (1994) points out two different classifications that can be made by individuals for quadrilaterals. One of these is the hierarchical classification made by relating the quadrilaterals as subsets according to their properties.

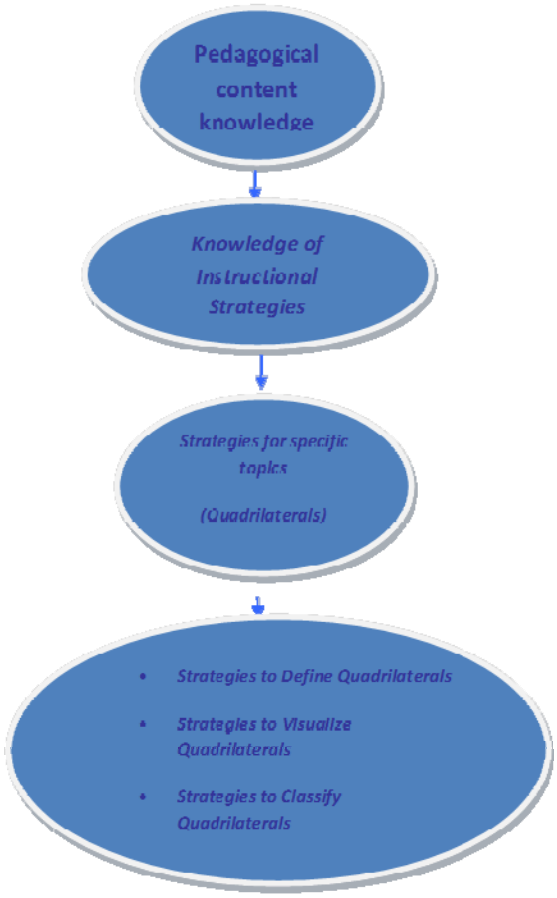


Figure 1. Theoretical Framework to be Used in the Study

Whereas the other is the partition classification which means classifying quadrilaterals into different sets according to their independent properties. De Villiers (1994) has stated that hierarchical classification makes familial relations more understandable.

When all the aforementioned theoretical structures effective in understanding and apprehending quadrilaterals are taken into account, it will be important during teaching how the teachers define and visualize quadrilaterals and how they structure familial relations. As a result, strategies for specific topics within the framework created by Magnusson et al. (1999) for the quadrilaterals have been examined under 3 sub-categories in this study; which are: strategies to define quadrilaterals, strategies to visualize quadrilaterals and strategies to classify quadrilaterals.

The theoretical framework synthesized above and presented in Figure 1 has been used in this study.

METHODS

The purpose of this study is to examine the middle school mathematics teachers' (10 to 13 year olds' teachers) teaching strategies of quadrilaterals. In this study, interview method has been preferred within the context of qualitative research.

Participants and setting

30 middle school mathematics teachers working at 12 different schools in the city of Izmir in Turkey have participated in this study. The teachers were selected based on the principle of voluntary. These teachers have 1 to 20 years of experience in their fields. The mathematics teachers who participated in this study have graduated from 4 years of mathematics teaching program in faculties of education. The teachers have graduated from these programs by taking the courses for major field of study as well as pedagogical knowledge and content knowledge. The topic of quadrilaterals has been examined in this study. Quadrilaterals are included in the 5 and 8th class curriculum at middle schools as well as the 5 and 7th class curriculum in Turkey. At these class levels, the topic of quadrilaterals included recognizing and understanding special quadrilaterals, drawing special quadrilaterals, determining their diagonals along with interior and exterior angles, measure-ment, forming area relations and solving problems.

Data collection

Interview method has been used for data acquisition. The questions that were asked to the teachers during interviews have been prepared in accordance with the pedagogical content knowledge teaching strategies component. The interview consisted of six questions which were intended to put forth the strategies that teachers use for defining, classification and visualization included in the strategies for specific topics sub-heading of the framework used for the study. These questions were reviewed by three field experts prior to being directed to the teachers and the required corrections were made. The questions of the interview which were prepared in such a manner were asked to a teacher and thus the pilot study for the interview questions was carried out. Two mathematics researchers who carried out the study reviewed the questions after the pilot interview thereby deciding on the final form of the questions.

The teacher was informed prior to the interview that the interview would be recorded. The interviews lasted in an average of 30 minutes. In addition to the data acquired during the interviews, the drawings of the teachers drawn during the interview were also used.

Data analysis

Content analysis was used to analyze the data acquired in the study. The basic process carried out in content analysis is to bring together the data that resembles each other within the framework of certain concepts and themes and to arrange and interpret these in a manner that will be understood by the reader (Simsek and Yildirim, 2006).

Data analysis was carried out in two stages. In the first stage, the voice records were analyzed. In the second stage, two researchers came together to determine the possible codes for the three strategies (defining, classification, visualization) used in the study. All these processes were carried out separately for each of the 30 teachers after which the analysis results were collected and reported.

RESULTS AND FINDINGS

Based on the analysis, the strategies used by teachers in

teaching quadrilaterals were coded as strategies for visualizing quadrilaterals, strategies for defining quadriclaterals and strategies for classification. The determined strategies were examined under these classifications.

Strategies to define quadrilaterals

Defining is an important aspect in learning quadrilaterals. Hence, the strategy of defining stands out in teaching quadrilaterals. The mathematics teachers have expressed in the interviews that they use strategies for defining quadrilaterals with visualization strategies when teaching quadrilaterals. When interviews were analyzed, it was determined that the teachers followed two basic methods while using definition strategy. One of these is the using of formal definition of quadrilaterals whereas the other is to define quadrilaterals by listing their properties.

Some of the teachers who use formal definition strategy have expressed during the interviews that they use the definitions given in textbooks. Whereas some have stated that they define quadrilaterals specifically themselves during the course and tell this definition to the students. When these definitions were examined it was observed that they were formal definitions. Statements by teachers who use formal definitions have been given below:

"I use the textbook and I make the students write down the definitions"
"It's actually based on the book, similar to the definition given in the book"
"I adhere to the textbook, making a definition is my priority."

As a result of the interviews carried out by the participants, it was determined that some teachers use a strategy in which they list the properties of the quadriclaterals instead of making a definition. It can be stated that teachers who follow this method generally use their own personal definitions. Statements by teachers who use this strategy have been given below:

".. I generally include properties within the definition"
".. We call out the properties one by one and count them together.."
"It is important to emphasize the properties and to list them when making a definition"
"We make a definition by using a few of the most distinct properties"

It was determined that some of the teachers who use this strategy emphasize only some of the properties of quadrilaterals, whereas others use the strategy by listing all the properties. The teachers who use the strategy to define by listing a certain number of properties generally use definitions such as "a shape with four equal sides" (for squares and rhombus), "a shape with opposite sides equal" (for rectangles and parallelograms] "a shape with opposite sides parallel" [for all quadrilaterals).

Mathematics teachers who participated in this study gave clues during the interviews as to how and in what way they used the formal definition strategy or the informal property listing strategy. Almost all teachers (except one) have indicated that they use these definition strategies in conjunction with visual representations. Some of the teachers have indicated that they use physical examples from their surroundings while some have stated that they use the models they draw on the board. Only one teacher has stated, "my priority is definition, I make the students draw after giving the definition" emphasizing that he has used the strategy to define quadrilaterals independent of drawing.

In addition, two different ways have been determined for using the definition strategy according to the statements of the teachers. One of these has been determined as the teacher defining quadrilaterals directly or listing their properties while the other consists of the teacher asking students to give a definition or list the properties themselves. Two different expressions by two different teachers have been given below regarding the use of definition strategy:

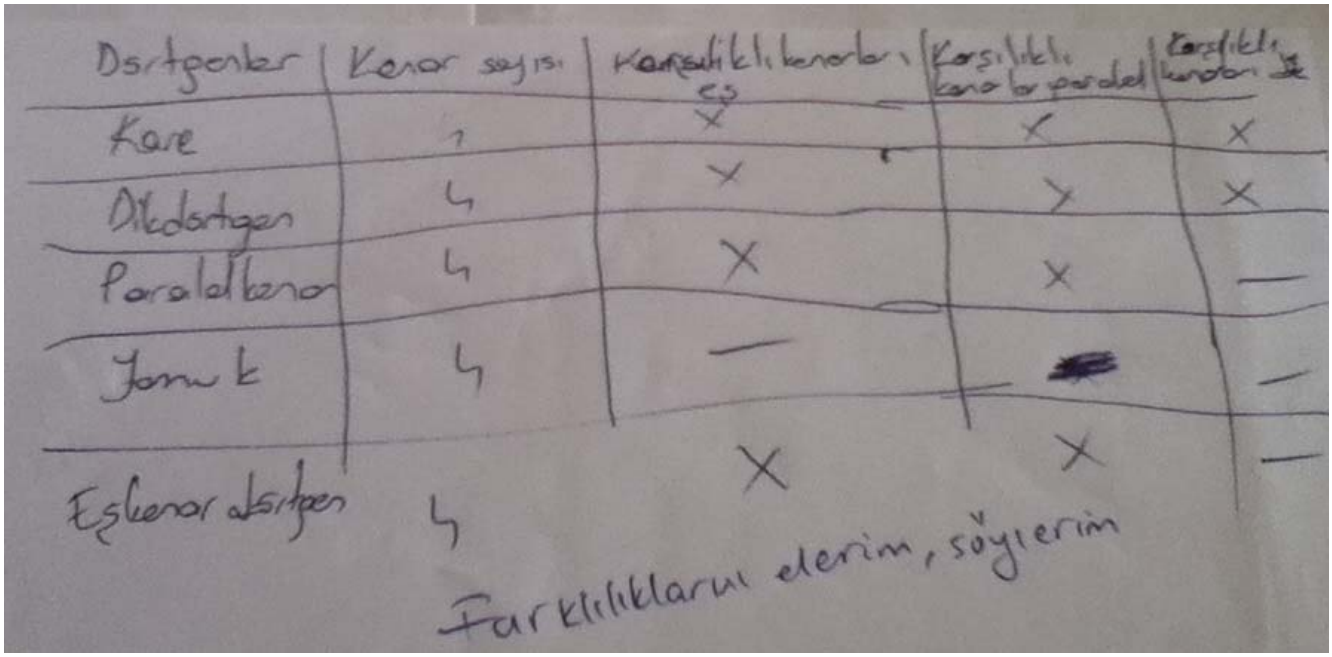
"I ask the students to define the quadrilaterals and they define the quadrilaterals by listing their properties"
"I first define the quadrilaterals one by one, after that I complete my definition by listing the properties of the quadrilateral".

Middle school mathematics teachers who participated in the study have generally used definition strategy together with visualization and have preferred asking the student directly or making the definition themselves.

Strategies to classify quadrilaterals

Another strategy used by the teachers for this topic has been determined as classifying quadrilaterals. Three different classification strategies have been determined as a result of the analysis of data acquired from the teachers. These are hierarchical classification, partial classification (used by De Villiers). The third one is the transformation classification which as has been named by the researchers. The teachers who use hierarchical classification have related quadrilaterals as sub-sets according to their properties. The teachers who use partial classification have used the different properties of quadrilaterals independent of each other to classify them. Whereas the teachers who use transformation classification have related quadrilaterals by transforming them into one another.

The teachers have generally stated that they use partial classification or transformation between quadrilaterals



Quadrilaterals	Number of sides	Opposite sides equal	Opposite sides parallel	Opposite sides right
Square	1	X	X	X
Rectangle	4	X	X	X
Parallelogram	4	X	X	-
Trapezoid	4	-	-	-
Rhombus	4	X	X	-

Figure 2. Drawing made by the Teacher Using Partial Classification

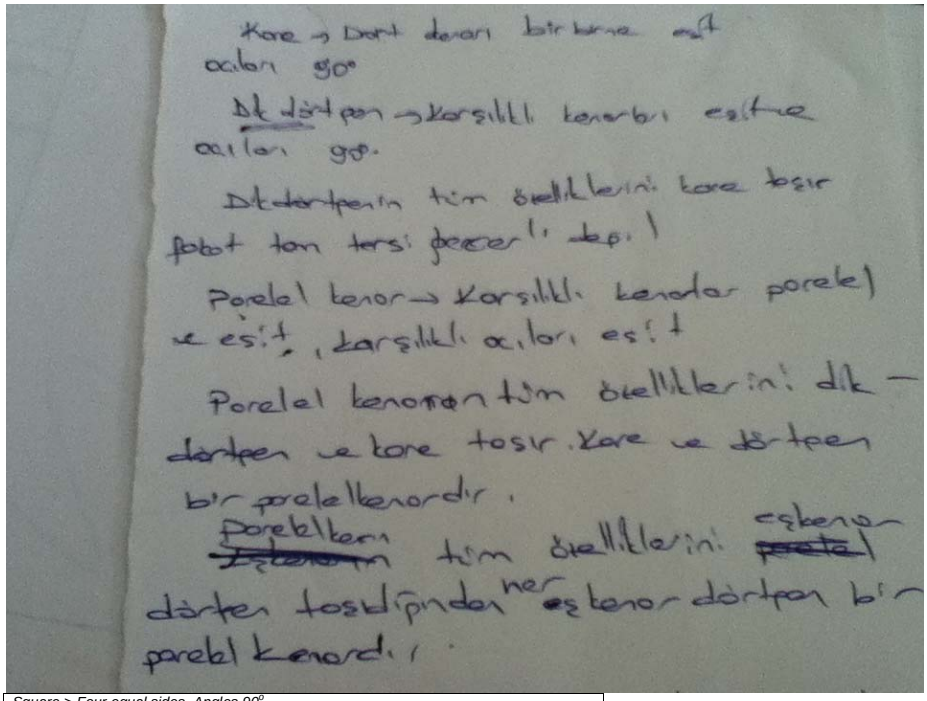
when teaching quadrilaterals. The general tendency observed among the teachers when making partial classification has been the expression of "we draw a table". When asked "What do you pay attention to when forming this table?" the teachers have generally replied by stating that this table is used to put forth the general differences between quadrilaterals. An example has been given in Figure 2 (English translation is given under the original table). The teachers who make a correct hierarchical classification take all relations between quadrilaterals into consideration and have carried out the classification by relating the quadrilaterals correctly as sub-sets. The classification of this teacher can be seen in Figure 3.

Finally, the commonly used classification strategy is making a classification by transforming the quadrilaterals into one another. The teachers have stated that they relate quadrilaterals by transforming them figuratively (changing the sides, cutting, bending, twisting). The dialogue carried out with a teacher who makes such a classification is given below and the figure drawn by this teacher can be seen in Figure 4;

"I form a parallelogram when I stretch a rectangle from its top right corner, I form a square when I cut it into two equal halves, the figure formed when I connect the rectangle from the mid-point of its sides is a rhombus and I obtain a trapezoid when I connect two points from the top side with the bottom. So in short, rectangle is the grandfather of this family."

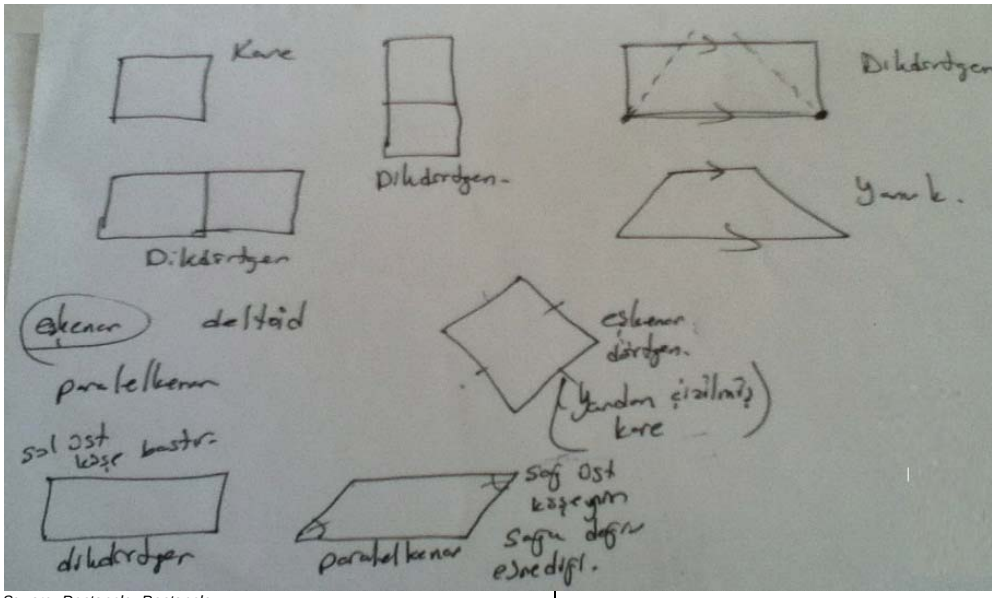
The classification drawing and explanations of the teacher who has made a classification similar to the one given above but who has changed sides and angles has been given in Figure 5;

The middle school mathematics teachers who participated in the study have generally explained their classification strategy by way of drawings and have preferred transforming quadrilaterals into each other as a strategy of classification. The interviews carried out have put forth that the teachers make some mistakes when classifying quadrilaterals. However, these have not been



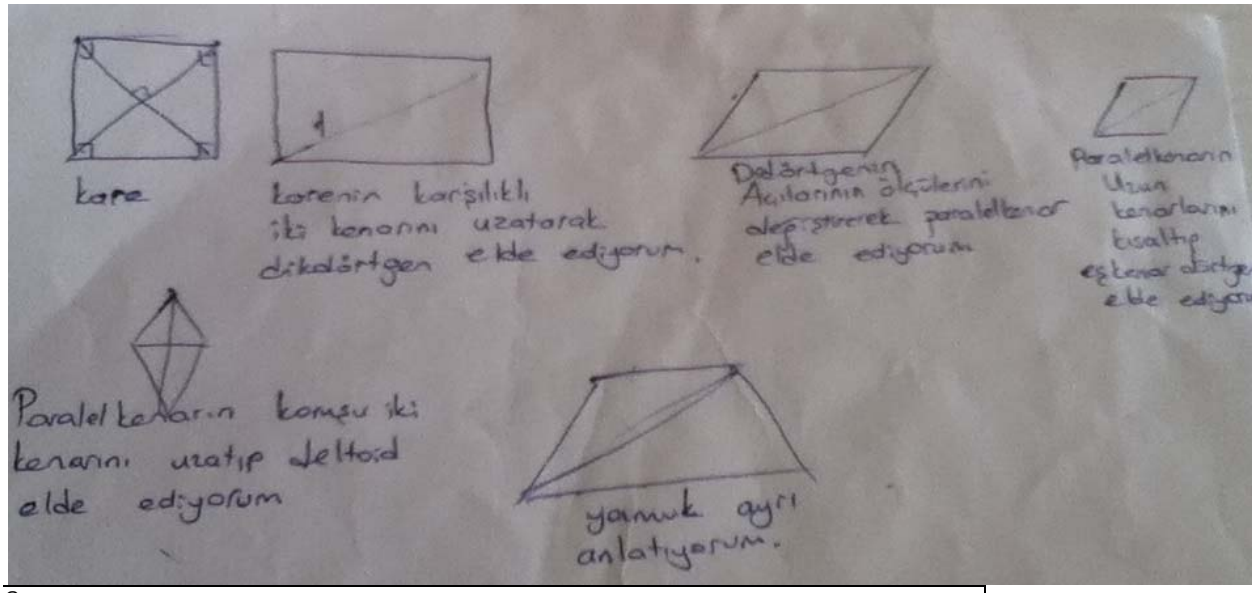
Square > Four equal sides. Angles 90°
Rectangle > Opposite sides equal. Angles 90°
Square has all the properties of a rectangle but the converse is not true
Parallelogram > Opposite sides parallel and equal. Opposite angles are equal
Rectangle and square has all the properties of a parallelogram. Square and rectangle are parallelograms.
Since a rhombus has the same properties of a parallelogram, every rhombus is also a parallelogram.

Figure 3. Drawing Made By Teacher Using Hierarchical Classification



Square, Rectangle, Rectangle
Rectangle, Trapezoid
Rhombus (Square drawn from the side)
Press from the top left corner: Rectangle
Parallelogram (Top right corner stretched towards the right)

Figure 4. Drawing Made By Teacher Using Quadrilateral Transformation Classification



Square
Rectangle is obtained by stretching the two opposite sides of a square.
Parallelogram is obtained by changing the angles of the rectangle.
The long sides of a parallelogram are shortened to obtain a rhombus
The neighboring sides of a parallelogram are stretched to obtain a deltoid.
I explain trapezoid separately.

Figure 5. Drawing made by the Teacher Who Uses Transformation Classification by Changing Sides and Angles

listed since it is out of the scope of the study.

Strategies to visualize quadrilaterals

Research participants have stated during the interviews that they use visual representations when teaching quadrilaterals. Visual representations contain the drawing models, the concrete models, etc, of quadrilaterals and have been defined as visualization strategies. Mathematics teachers have generally used visualization strategies when first mentioning a topic, when making a definition or when listing properties. Visualization strategies have been examined under three sub-categories. These are; visualization using examples from daily life, visualization using materials and visualization by drawing. Middle school mathematics teachers who participated in the study have stated that they use one or more of these strategies when teaching quadrilaterals.

Some participants have stated that they give examples from their surroundings such as door, window, kite, etc, to define quadrilaterals. Whereas some have stated that they visualize quadrilaterals by way of materials such as geometry board, pattern blocks, etc. Below, expressions of the teachers who use these two visualization strategies have been given as examples:

"I try to start by examples as much as I can and I give examples from daily life. I bring a geometry board to the classroom."
"I ask the students to give examples of quadrilaterals around us; like doors, windows, floor of the classroom, etc."
"We cut A4 paper and use pattern blocks and rubbers to form the quadrilateral."

Almost all the teachers who participated in the study have stated that they use drawings when teaching this topic. According to the analysis results obtained from the answers of the teachers, it has been determined that the teachers pay attention to two strategies for drawings. These are using tools to draw the quadrilateral properly and using the hand to draw the quadrilateral without any tool. Some of the teachers who use drawing tools and who give importance to drawing the quadrilateral in accordance with its properties have emphasized that they do not accept drawings made without using rulers. Whereas some teachers have stated that they use graph notebooks to make drawings.

Another result obtained from the analysis of the answers given by the teachers for quadrilateral drawing is that it is important to "draw the quadrilateral correctly". The teachers have defined the correct drawing of a quadrilateral as realizing the distinctive features of quadrilaterals when faced with other quadrilaterals. In addition, some teachers have stated that it is important to

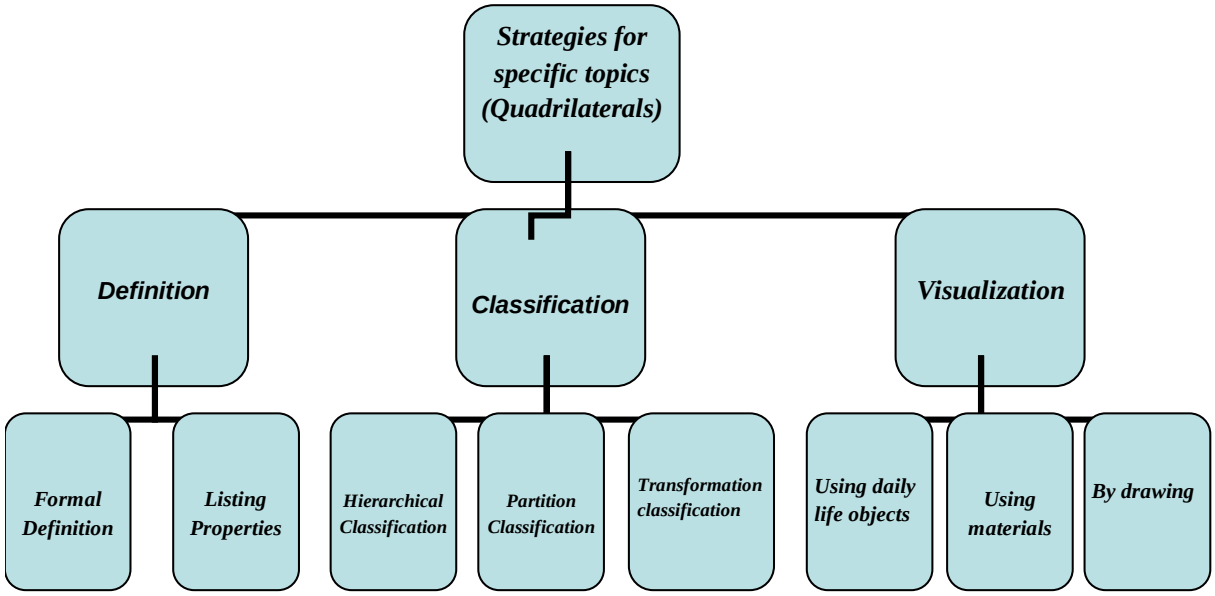


Figure 6. Strategies Acquired for Quadrilateral Teaching

introduce the quadrilateral to students by drawing it in different positions. This result indicates that the teachers can move away from typical prototype drawings and use the critical properties of quadrilaterals in their drawings.

In this study, emphasis has been given on "teaching strategies" component. The data acquired has enabled us to give details on special teaching strategies of quadrilaterals. All strategies obtained as a result of the study have been given in Figure 6.

DISCUSSIONS, CONCLUSIONS AND IMPLICATIONS

This study has examined the pedagogical content knowledge of the teachers within the context of "quadrilateral specific strategies" by carrying out interviews. Quadrilateral specific strategies have been classified as defining quadrilaterals, classifying quadrilaterals and visualizing quadrilaterals. The study results have put forth that the teachers use these three strategies one by one or in unison. The size of the study group is not suited for various generalizations. However, since this study was carried out with the teachers who teach at different schools and different grades (5^{th} and 7^{th} Grade), it is possible to make some distinctions about the strategies used for teaching quadrilaterals.

It has been determined that the definition strategy is used together with visualization strategy. Informal definitions are used in addition to formal definitions and that in general a personal definition is given by listing the various properties of quadrilaterals. When studies on understanding quadrilaterals and the concept image are examined, it has been observed that individuals define geometric concepts by way of an image they form of the concept and that they use personal definitions instead of formal ones (Sarfaty and Patkin, 2013; Turnuklu et al., 2013; Hershkowitz, 1989; Burger and Shaughnessy, 1986). To this end, it can be stated that the definitions used by mathematics teachers who participated in this study are shaped pursuant to their perceptions. A more detailed study is required to put forth why they go out of formal definitions. In addition, the results obtained in this study putting forth that the teachers find it more effective to have the students make the definition instead of the teacher expressing the definition directly is in accordance with literature data (De Villiers, 1998; Walcott et al., 2009).

The teachers who participated in this study have generally used partial classification of quadrilaterals and transformation classification of quadrilaterals. According to De Villiers, hierarchical classification is more effective. Because it is stated that hierarchical classification is important in geometrical thinking and problem solving (De Villiers, 1998; 1994). In addition, the study results indicate that the teachers use classification strategy by transforming quadrilaterals into one another. The teachers perceive relating quadrilaterals and making classifications as "transformation". No study has been found in literature which examines classification between quadrilaterals as transformation [changing quadrilaterals formally (changing the sides, cutting, folding, twisting, etc.)].

The study has been carried out with 30 middle school mathematics teachers. A study can be carried out with a larger group of teachers in order to determine different strategies that the teachers use in teaching quadrilaterals or make generalization the strategies. In addition, making

observations in classes to determine how these strategies are used can provide a more depth and rich data. Various other studies can be carried out to find how effective these strategies are. In addition, the strategies used in the teaching of quadrilaterals are used for educating teacher candidates.

In this study, strategies for specific topics of teaching strategies, component of pedagogical content knowledge has been examined. Different studies can be carried out on different pedagogical content knowledge components to examine the relationship between the results.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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