# Present and future value formulae for uneven cash flow based on the performance of a business 

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#### Abstract

Business firm's income is not constant or fixed from period to period. Because of this, a firm's cash inflow or out flow is uneven. The decision of a firm either to invest or to borrow from creditors based on uneven cash in-flow need to have a future or a present value prediction formula. The problem to find future and present value formulae for uneven cash flow stayed unsolved for long period of time (Pandey, 1999). However, in this paper, I want to show future and present value of uneven cash flow prediction formulae based on the performance rate $\left(P_{n}\right)$ of a business. The firm cash out flow either for investment or for repayment of the borrowed loan growth according to the performance rate ( $p$ ) of the firm. The Performance rate $\left(P_{n}\right)$ is a percentage by which the current performance of the business exceeds the previous. Therefore, the firm cash out flows either for investment or for repayment of the borrowed loan growth according to the performance rate (p) of the firm.


Key words: Uneven cash flow, present value formula, future value formula, performance rate, nonperforming loan.

## INTRODUCTION

Money does have different value at different time period. The money we owe now is worth more than we need to owe in the future. The facts behind this idea are: we prefer to use money more productively now to get a real return in the future, during inflation period, the money we use now does have more purchasing power than the money we use in the future, and at last, since the future is full of risk and uncertainty, we prefer current consumption to future consumption. This phenomenon is also referred to as time preference of money which is expressed by an interest rate (Damodaran, 2001; Pandey, 1999; Myddelton, 1995).
An interest rate is defined as the price paid by a borrower to bank/ creditor for the use of money/ resources during the given time period. A borrower from a bank pays the principal and a percentage of the principal per unit of time.
A money depositor into the bank account will receive at the end of interest period, the principal deposit amount and a percentage of the principal per unit of time which is the interest amount.
Compounding and discounting techniques are the two
methods by which time value of money can be calculated. The future and present values of annuity formulae developed in such technique is only valid for equal amount of periodic cash flow. The formulae have the following shortcomings:

1. The formulae do not calculate a cash flow which grows according to the increase of the project's performance.
2. The formulae do consider a project performance to be same throughout the project life, which is impossible. A project whose performance does not change according to the market situation throughout the project life may be out of the market.
3. The formulae do not consider the future inflation period where the project cash inflow will be reduced.
4. The formulae do not consider flexibility of deposit to bank or flexibility of payment to creditors.
5. The present value formula does not consider low cash flow at initial stage.
At initial stage of investment where there is high burden of investment cost, high expense for advertisement of the business products, there might be no cash inflow for the
firm. As a result of this, the set repayment using present value formula by creditors without considering the business firm's initial investment cost becomes a burden for a borrower.
6. The repayment set for borrowers by using annuity formula decreases from high burden to low because the repayment amount in the end of the loan period comes down to small amount and the borrower gets relief (Pandey, 1999). This is meaningless since the borrower should have gotten a relief of loan payment at the beginning of loan period where the project cash inflow is very low.
7. The cash flow we are going to calculate by the present value formula does not have any future risk premium though the interest rate does have, and this creates a mismatch between the cash flow and the interest rate.

As have seen earlier, present and future value formulae fail to calculate uneven cash flow. In the investment decision of a firm, one could receive uneven cash inflow which does not enable him/her to make a decision because there is no formula to calculate uneven cash investment shortly. As a result of this, we are forced to use a laborious technique (Pandey, 1999; Mosich, 1963; Chandra, 1993) by which we are going to calculate each period's cash flow to get the total sum.
In this paper, it is shown how the sum of uneven cash flow either for investment or for repayment of borrowed loan from bank based on performance of the business firm can be determined. The present value formula gives an advantage to borrowers since it relieves borrowers' suffering from paying huge amount at the beginning periods. The firm at the initial stage is assumed at high establishment cost and eventually, after covering the investment cost, the loan repayment is assumed to rise according to the investment performance rate. The repayments calculated and set for each loan periods by creditors/banks in such a way will benefit them in collection of loan interest since at the beginning years (periods) the principal loan amount is not much affected by the small repayments compared to the next period repayments for each of which it was assumed that the interest amount should be cleared first and the principal excess amount which is above the interest amount.
On the other hand, future value formula relaxed depositors for they can deposit each periods excess above their consumption. A newly entrant business firm into market does have a little amount of cash at initial stage and eventually after covering the establishment cost the liquidity amount on hand grows accordingly. The growth of the cash inflow of the business firm depends on its performance. When performance of the business improves, its profit and cash inflows increase in the same way. As a result of this, the depositor can deposit the excess above his/her business consumption, starting with a small amount of money which can eventually increase from period to period to a higher amount according to the business performance.

Although, traditional accounting measures of performance of a firm are profit, earning per share (EPS), return on investment (ROI),free cash flow (FCF),capital productivity (KP), labor productivity (LP) and return on capital employed (ROCE) each of each ignores cash and cost of capital so as to generate the target profit (Brealey et al, 2003; Chandra, 1993). Rather, the best measuring tool of performance of a business firm is economic value added which provides the money value created for investor in a given period of time by weighting the profit generated against the cost of capital employed so as to generate that profit Firer et al. (2004). From this, since EVA considers the amount of capital invested, the return earned on capital and the cost of capital (WACC) reflecting the risk adjusted required rate of return, it is thought to have all characteristics of the measure though it is valid only for short period of time.
Furthermore, since EVA (Economic Value Added) is considered as a measure of both performance and value of a business firm, it is assumed to be a way to determine the value created excess above the required return for the shareholder of the business firm. The firm creates wealth for the shareholder when the revenue of the firm exceeds over the cost of doing business and the cost of capital. A business firm creates value for its shareholders on the bases of positive EVA rather than simply making accounting profits. The positive magnitude of EVA indicates as the business firm is improving its net cash return on invested capital. The increment of EVA of the firm from year to year will result an increase of the market value of the firm (Damodaran, 2001).
The existence of accounting information of a firm for a single accounting period helps the manager to grasp the basic know how of the firm performance in that accounting period. A manager who has good experience of the firm performance helps in facilitating to predict future performance of a firm basing on the past financial statement such as income statement and balance sheet. The availability of past trend records help to calculate and predict progressive performance rate of the firm so as to determine progressive cash inflow on the firm investment return and the firm progressive bank loan repayment (Alexander Hamilton Institute, 1998; Reilly and Schweihs, 2000). Since the firm's performance rate is assumed to be progressive, the cash flow of the firm assumes to grow from period to period. Performance rate is a percentage by which the current performance of a firm (in this case, EVA) growth from the previous.

## Research objective

The objective of this research is to introduce future and present value formulae for a series of periodic cash flow (payment or receipts) of unequal amount.

## METHODOLOGY

A business firm's decision to borrow money from bank or creditor
often involves receiving cash now in exchange for a promise to make payment in the agreed period.

An investor who is going to borrow some fixed amount of money will get into commitment to pay the loan repayment amount at every period as per the contractual agreement between the investor and creditor. The loan repayment amount pre-calculated and set by the creditors assuming to pay the period interest amount and a portion of the principal by the excess above the interest amount. Each of the next period repayment amounts is a product of the preceding repayment amount, and of the current project performance rate.

At the initial repayment period, the repayment amount is assumed to be a small fixed amount without performance rate. However, in the next repayment period after the first period, repayment amount increased according to the growth of business performance.
Assume that the first small repayment amount is $a_{1}$ and the next payment after the first repayment consequently be calculated as $a_{1}\left(P_{1}+1+i\right)\left(P_{2}+1+i\right) \ldots .\left(P_{n-2}+1+i\right) P_{n-1}$. Such that repayment period and amount are presented as:
period $1=a_{1}$; period $2=a_{1}\left(P_{1}+1+i\right)$; period $3=$ $a_{1}\left(P_{1}+1+i\right) P_{2} ; \ldots \ldots ;$ period $\mathrm{n}=$
$a_{1}\left(P_{1}+1+i\right)\left(P_{2}+1+i\right) \ldots\left(P_{n-2}+1+i\right) P_{n-1}$
Where $P_{n}=$ Performance rate, any arbitrary figure greater than zero, which can be measured as the percentage by which the $(n+1)^{\text {th }}$ period EVA growth from the $\mathrm{n}^{\text {th }}$ period EVA, that is:

$$
P_{n}=\frac{E V A_{n+1}-E V A_{n}}{E V A_{n}} .
$$

The reason why the author preferred to put performance rate in such a way is to save the cash flow from unnecessary exaggeration because the ratio is very small, though the $n^{\text {th }}$ period performance rate is greater than/or less than the $(n-1)^{\text {th }}$ period performance rate. Here, EVA for $n$ periods is to mean projected net income for n loan periods on the firm projected income statement. i = bank interest rate which is added to the performance rate since the $\mathrm{n}^{\text {th }}$ period repayment should contain a portion of the principal amount and of the period interest amount (Alexander Hamilton Institute, 1998). It is between 0 and $1 ; \mathrm{n}=$ loan period.

The current cash value of the future cash flow of these loan repayments can be expressed as follow:

$$
\begin{gathered}
{\left[\prod_{m=1}^{n-1}\left(P_{m}+1+i\right)+(i+1)^{n-1}\right]\left(\frac{a_{1}}{(1+i)^{n}}\right)=\frac{a_{1}}{(i+1)}+\frac{a_{1}\left(P_{1}+1+i\right)}{(1+i)^{2}}+\frac{a_{1}\left(P_{1}+1+i\right) P_{2}}{(1+i)^{3}}+\frac{a_{1}\left(P_{1}+1+i\right)\left(P_{2}+1+i\right) P_{3}}{(1+i)^{4}}+\ldots \ldots \ldots \ldots . .} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots+\ldots \ldots\left(P_{1}+1+i\right)\left(P_{2}+1+i\right)\left(P_{3}+1+i\right) \ldots\left(P_{n-2}+1+i\right) P_{n-1} \\
(1+i)^{n}
\end{gathered}
$$

Another decision of a business firm is a decision to invest cash now in order to receive cash, goods or services in the future period. Here also, let us represent each period and investing cash as:
$\left.\mathrm{T}_{2}\right) ; \ldots .$. period $\mathrm{n}=\mathrm{a}_{1} \mathrm{a}_{1}$
$\left.\left(p_{1}+1\right)\left(p_{2}+1\right) \ldots\left(p_{n-3}+1\right)\left(p_{n-2}+1\right)\right)\left(\mathrm{W}_{\mathrm{n}-1}-\mathrm{T}_{\mathrm{n}-1}\right)$
period $1=\mathrm{a}_{1} ;$ period $2=\mathrm{a}_{1}\left(\mathrm{~W}_{1}-\mathrm{T}_{1}\right)$; period $\left.3=\mathrm{a}_{1}\left(p_{1}+1\right)\right)\left(\mathrm{W}_{2}-\right.$
Where

$$
W_{n}=P_{n}+1, \mathrm{~T}_{\mathrm{n}}=1-\left(P_{n}+1\right)(i) \text { for }\left(\mathrm{P}_{\mathrm{n}}+1\right)(\mathrm{i}) \text { between } 0 \text { and } 1
$$

(that is, $\left.0<\left(P_{n}+1\right)(i)<1\right) ; T_{n}=$ the firm performance other than bank's deposit interest. The deposit interest rate should not be included into the project performance since it is always less than the bank lending interest rate; $\mathrm{i}=$ bank interest rate which is between 0 and 1; $\mathrm{n}=$ investing period; $W_{n-1}-T_{n-1}=$ a portion of performance
rate by which the excess above consumption should be saved into a bank account
Therefore, the future value of investing cash is now represented as follows:

$$
\begin{aligned}
a_{1}(i+1)\left[\prod_{m=1}^{n-1}\left(p_{m}+1\right)(1+i)^{n}-i\right]= & a_{1}(i+1)+a_{1}\left(W_{1}-T_{1}\right)(i+1)^{2}+a_{1}\left(p_{1}+1\right)\left(W_{2}-T_{2}\right)(i+1)^{3}+\ldots \ldots \ldots \ldots \\
& \ldots \ldots \ldots \ldots . .+a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right) \ldots\left(\mathrm{p}_{\mathrm{n}-2}+1\right)\left(W_{n-1}-T_{n-1}\right)(i+1)^{n}
\end{aligned}
$$

Where:

$$
a_{1}=a_{1}, a_{2}=p_{1} a_{1}, \mathrm{a}_{3}=p_{2}\left(a_{1}+a_{2}\right), \mathrm{a}_{4}=p_{3}\left(a_{1}+a_{2}+a_{3}\right), \ldots \ldots \ldots, \mathrm{a}_{\mathrm{n}}=p_{n-1}\left(a_{1}+a_{2}+\ldots .+a_{n-1}\right)
$$

Such that:

$$
\begin{gathered}
a_{1}=a_{1}, a_{2}=p_{1}\left(a_{1}\right), \mathrm{a}_{3}=p_{2}\left(a_{1}\right)\left(p_{1}+1\right), \mathrm{a}_{4}=p_{3}\left(a_{1}\right)\left(p_{1}+1\right)\left(p_{2}+1\right), \ldots . . \\
\ldots ., \mathrm{a}_{\mathrm{n}}=p_{n-1}\left(a_{1}\right)\left(p_{1}+1\right)\left(p_{2}+1\right) \ldots\left(p_{n-2}+1\right)
\end{gathered}
$$

Let the $n^{\text {th }}$ period cash flow $\left(a_{n}\right)$, performance rate $\left(p_{n}\right)$ and the discount rate $(x)$ be expressed by any figure which can mathematically be expressed as $a_{n} \in R$ (Real numbers) for all $n \in N$ (Natural numbers). Such that $a_{n}, x, p \in R$. (where, $\in$ stands for elements). Then the sum of cash flow for n periods can be explained as:

$$
\sum_{m=1}^{n+1} a_{m}=a_{1}+a_{2}+\ldots \ldots \ldots \ldots .+a_{n+1}
$$

This can be expressed as:

$$
\sum_{m=1}^{n+1} a_{m}=\frac{a_{1} x}{x}+\frac{a_{2} x^{2}}{x^{2}}+\ldots \ldots \ldots . .+\frac{a_{n+1} x^{n+1}}{x^{n+1}}, \text { for } \mathrm{x} \neq 0
$$

Split the terms as:

$$
\begin{aligned}
= & \left.\frac{1}{x}\left(a_{1} x+0\right)+\frac{1}{x^{2}}\left(\left(a_{2} x^{2}+a_{1} x\right)-\left(a_{1} x\right)\right)+\frac{1}{x^{3}}\left(\left(a_{3} x^{3}+a_{2} x^{2}+a_{1} x\right)\right)-\left(a_{2} x^{2}+a_{1} x\right)\right)+\ldots \ldots \\
& \ldots \ldots \ldots \ldots \ldots . . . . . . . . \\
x^{n+1} & \left(\left(\mathrm{a}_{\mathrm{n}+1} x^{n+1}+a_{n} x^{n}+\ldots . .+a_{1} x\right)-\left(a_{n} x^{n}+a_{n-1} x^{n-1}+\ldots .+a_{1} x\right)\right)
\end{aligned}
$$

Collecting like terms in the brackets, we get:

$$
\begin{aligned}
& =a_{1} x\left(\frac{1}{x}-\frac{1}{x^{2}}\right)+\left(\mathrm{a}_{2} x^{2}+a_{1} x\right)\left(\frac{1}{x^{2}}-\frac{1}{x^{3}}\right)+\ldots \ldots \ldots \ldots . .+\left(a_{n} x^{n}+a_{n-1} x^{n-1}+\ldots . .+a_{1} x\right)\left(\frac{1}{x^{n}}-\frac{1}{x^{n+1}}\right)+ \\
& \frac{1}{\mathrm{x}^{\mathrm{n+1}}}\left(a_{n+1} x^{n+1}+a_{n} x^{n}+\ldots . .+a_{1} x\right) \\
& =\left(a_{1} x\right)\left(\frac{1}{x}-\frac{1}{x^{2}}+\frac{1}{x^{2}}-\frac{1}{x^{3}}+\ldots+\frac{1}{x^{n}}-\frac{1}{x^{n+1}}\right)+\left(a_{2} x^{2}\right)\left(\frac{1}{x^{2}}-\frac{1}{x^{3}} \ldots .+\frac{1}{x^{n}}-\frac{1}{x^{n+1}}\right)+\ldots \ldots \ldots \ldots . .+\left(a_{n} x^{n}\right)\left(\frac{1}{x^{n}}-\frac{1}{x^{n+1}}\right)+ \\
& \frac{1}{\mathrm{x}^{\mathrm{n}+1}}\left(a_{n+1} x^{n+1}+a_{n} x^{n}+\ldots . .+a_{1} x\right) \\
& =a_{1} x\left(\frac{1}{x}-\frac{1}{x^{n+1}}\right)+a_{2} x^{2}\left(\frac{1}{x^{2}}-\frac{1}{x^{n+1}}\right)+\ldots \ldots \ldots . .+a_{n} x^{n}\left(\frac{1}{x^{n}}-\frac{1}{x^{n+1}}\right)+\frac{1}{x^{n+1}}\left(a_{n+1} x^{n+1}+a_{n} x^{n}+\ldots .+a_{1} x\right) \\
& =a_{1} x\left(\frac{x^{n}-1}{x^{n+1}}\right)+a_{2} x^{2}\left(\frac{x^{n-1}-1}{x^{n+1}}\right)+\ldots \ldots \ldots \ldots \ldots \ldots . . a_{n} x^{n}\left(\frac{x-1}{x^{n+1}}\right)+\frac{1}{x^{n+1}}\left(a_{n+1} x^{n+1}+a_{n} x^{n}+\ldots .+a_{1} x\right)
\end{aligned}
$$

Multiply both sides by $\frac{x^{n+1}}{x-1}$, for $\mathrm{x} \neq 1$ :

$$
\left(\sum_{m=1}^{n+1} a_{m}\right)\left(\frac{\mathrm{x}^{\mathrm{n}+1}}{x-1}\right)=a_{1} x \frac{\left(x^{n}-1\right)}{x-1}+a_{2} x^{2} \frac{\left(x^{n-1}-1\right)}{x-1}+\ldots \ldots .+a_{n} x^{n} \frac{(x-1)}{x-1}+\left(\frac{1}{x-1}\right)\left(\mathrm{a}_{\mathrm{n}+1} x^{n+1}+a_{n} x^{n}+\ldots .+a_{1} x\right)
$$

Since the formula of geometric progression at ratio $=x$ can be
expressed as $\frac{x^{n}-1}{x-1}=1+x+\ldots .+x^{n-1} \quad([12])$, it follows that:

$$
=a_{1}\left(x+x^{2}+\ldots . .+x^{n}\right)+a_{2}\left(x^{2}+x^{3}+\ldots \ldots . . x^{n}\right)+\ldots \ldots+a_{n}\left(x^{n}\right)+\left(\frac{1}{x-1}\right)\left(a_{1} x+a_{2} x^{2}+\ldots .+a_{n+1} x^{n+1}\right)
$$

Equivalently, this can be expressed as:

$$
=\left(a_{1}\right) x+\left(a_{1}+a_{2}\right) x^{2}+\left(a_{1}+a_{2}+a_{3}\right) x^{3}+\ldots \ldots .+\left(a_{1}+a_{2}+\ldots .+a_{n}\right) x^{n}+\left(a_{1} x+a_{2} x^{2}+\ldots .+a_{n+1} x^{n+1}\right)\left(\frac{1}{x-1}\right)
$$

Let $a_{n+1}=0$, then it follows that:

$$
\begin{equation*}
\left(\sum_{m=1}^{n} a_{m}\right)(x)^{n}\left(\frac{x}{x-1}\right)=\left(a_{1} \frac{x}{x-1}\right)(\mathrm{x})+\left(a_{1}+a_{2} \frac{x}{x-1}\right)(\mathrm{x})^{2}+\left(a_{1}+a_{2}+a_{3} \frac{x}{x-1}\right)(\mathrm{x})^{3}+\ldots . .+\left(a_{1}+a_{2}+\ldots .+a_{n} \frac{x}{x-1}\right)(\mathrm{x})^{n} \tag{1}
\end{equation*}
$$

From this expression, let us assume that the cash flow of a project for each period can be defined as:

$$
a_{n}=p_{n-1}\left(a_{1}+a_{2}+a_{3}+\ldots .+a_{n-1}\right)
$$

which can be written as:

$$
a_{1}=a_{1}, a_{2}=p_{1} a_{1}, \mathrm{a}_{3}=p_{2}\left(a_{1}+a_{2}\right), \mathrm{a}_{4}=p_{3}\left(a_{1}+a_{2}+a_{3}\right), \ldots \ldots \ldots, \mathrm{a}_{\mathrm{n}}=p_{n-1}\left(a_{1}+a_{2}+\ldots+a_{n-1}\right)
$$

or can be written as:

$$
\begin{gathered}
a_{1}=a_{1}, a_{2}=p_{1}\left(a_{1}\right), \mathrm{a}_{3}=p_{2}\left(a_{1}\right)\left(p_{1}+1\right), \mathrm{a}_{4}=p_{3}\left(a_{1}\right)\left(p_{1}+1\right)\left(p_{2}+1\right), \ldots . \\
\ldots, \mathrm{a}_{\mathrm{n}}=p_{n-1}\left(a_{1}\right)\left(p_{1}+1\right)\left(p_{2}+1\right) \ldots\left(p_{n-2}+1\right)
\end{gathered}
$$

such that, $P_{n+1} \leq$ or $\geq P_{n} \& P_{0}=1$ and putting this in (1), we have:

$$
\begin{aligned}
& =\left(a_{1} \frac{x}{x-1}\right) x+\left(a_{1}+a_{1} P_{1} \frac{x}{x-1}\right) x^{2}+\left(\left(a_{1}+a_{2}\right)+\left(a_{1}+a_{2}\right) P_{2} \frac{x}{x-1}\right) x^{3}+ \\
& \left(\left(a_{1}+a_{2}+a_{3}\right)+\left(a_{1}+a_{2}+a_{3}\right) P_{3} \frac{x}{x-1}\right) x^{4}+\ldots \ldots \ldots+\left(\left(a_{1}+a_{2}+a_{3}+\ldots . .+a_{n-1}\right)+\left(a_{1}+a_{2}+\ldots \ldots+a_{n-1}\right) P_{n-1} \frac{x}{x-1}\right) x^{n} \\
& \quad=a_{1} \frac{x^{2}}{x-1}+\left(a_{1}\right)\left(\left(p_{1}+1\right) x-1\right) \frac{x^{2}}{x-1}+\left(a_{1}+a_{2}\right)\left(\left(P_{2}+1\right) x-1\right) \frac{x^{3}}{x-1}+\left(a_{1}+a_{2}+a_{3}\right)\left(\left(P_{3}+1\right) x-1\right) \frac{x^{4}}{x-1}+\ldots \\
& \quad \ldots \ldots \ldots \ldots .+\left(a_{1}+a_{2}+a_{3}+\ldots . .+a_{n-1}\right)\left(\left(P_{n-1}+1\right) x-1\right) \frac{x^{n}}{x-1}
\end{aligned}
$$

Multiplying both sides of the equation by $\mathrm{x}-1$, we obtain:

$$
\begin{gathered}
\left(\sum_{m=1}^{n} a_{m}\right)(x)^{n+1}=a_{1} x^{2}+a_{1}\left(\left(p_{1}+1\right) x-1\right) \mathrm{x}^{2}+\left(a_{1}+a_{2}\right)\left(\left(\mathrm{p}_{2}+1\right) x-1\right) x^{3}+\left(a_{1}+a_{2}+a_{3}\right)\left(\left(p_{3}+1\right) x-1\right) x^{4}+\ldots \ldots \\
\ldots \ldots \ldots \ldots \ldots \ldots . .+\left(a_{1}+a_{2}+\ldots .+a_{n-1}\right)\left(\left(\mathrm{p}_{\mathrm{n}-1}+1\right) \mathrm{x}-1\right) x^{n}
\end{gathered}
$$

$$
\begin{align*}
& \left(\sum_{m=1}^{n} a_{m}\right)(x)^{n+1}=a_{1} x^{2}+a_{1}\left(\left(p_{1}+1\right) x^{3}-x^{2}\right)+\left(a_{1}+a_{2}\right)\left(\left(\mathrm{p}_{2}+1\right) x^{4}-x^{3}\right)+\left(a_{1}+a_{2}+a_{3}\right)\left(\left(p_{3}+1\right) x^{5}-x^{4}\right)+\ldots \ldots \\
& .+\left(a_{1}+a_{2}+\ldots .+a_{n-1}\right)\left(\left(\mathrm{p}_{n-1}+1\right) \mathrm{x}^{n+1}-x^{n}\right) \tag{2}
\end{align*}
$$

If the bank interest rate (i) is expressed as $x=\frac{1}{i+1}$ and putting $\quad$ this in (2) and multiplying both sides of the equation by ( $i+1$ ) we have the following:

$$
\begin{gather*}
{\left[\sum_{m=1}^{n} a_{m}\right]\left(\frac{1}{i+1}\right)^{n}=\frac{a_{1}}{(i+1)}+\frac{a_{1}\left(p_{1}-i\right)}{(i+1)^{2}}+\frac{\left(a_{1}+a_{2}\right)\left(p_{2}-i\right)}{(i+1)^{3}}+\frac{\left(a_{1}+a_{2}+a_{3}\right)\left(p_{3}-i\right)}{(i+1)^{4}}+\ldots \ldots \ldots \ldots} \\
\ldots \ldots \ldots \ldots \ldots \ldots+\frac{\left(a_{1}+a_{2}+\ldots \ldots+a_{n-1}\right)\left(p_{n-1}-i\right)}{(i+1)^{n}} \tag{3}
\end{gather*}
$$

Suppose that in equation (3), $P_{n}=P_{n}+i$, and adding both sides
of the equation $\frac{a_{1}}{(i+1)}$ we obtain the following present value formula:

$$
\begin{gather*}
{\left[\prod_{m=1}^{n-1}\left(P_{m}+1+i\right)+(i+1)^{n-1}\right]\left(\frac{a_{1}}{(1+i)^{n}}\right)=\frac{a_{1}}{(i+1)}+\frac{a_{1}\left(P_{1}+1+i\right)}{(1+i)^{2}}+\frac{a_{1}\left(P_{1}+1+i\right) P_{2}}{(1+i)^{3}}+\frac{a_{1}\left(P_{1}+1+i\right)\left(P_{2}+1+i\right) P_{3}}{(1+i)^{4}}+\ldots \ldots \ldots \ldots . .} \\
\ldots \ldots \ldots \ldots \ldots . . \ldots \frac{a_{1}\left(P_{1}+1+i\right)\left(P_{2}+1+i\right)\left(P_{3}+1+i\right) \ldots\left(P_{n-1}+1+i\right) P_{n}}{(1+i)^{n}} \tag{4}
\end{gather*}
$$

The initial cash flow (a1), in equation (4), growth moves progressively from one period to another by multiplying with progressive performance rate of the period. Each of the period's cash flow contains an interest rate that resists inflation and risk which might be happened in the future period. Furthermore each of the period cash flow contains the period performance rate that can
move along with the strength of the business which can lead the cash flow highly volatile.

Again, let us consider equation (2), by putting first, $x=(i+1)$ and then deducting $a_{1}(i+1)^{2}$ from both sides, and then after adding both side of the equation $a_{1}(i+1)$, we have the following future value formula:

$$
\begin{gather*}
{\left[\sum_{m=1}^{n} a_{m}\right](i+1)^{n+1}-a_{1}(i+1) i=a_{1}(i+1)+a_{1}\left(W_{1}-T_{1}\right)(i+1)^{2}+\left(a_{1}+a_{2}\right)\left(W_{2}-T_{2}\right)(i+1)^{3}+\ldots \ldots \ldots \ldots} \\
\ldots \ldots \ldots \ldots . .+\left(\mathrm{a}_{1}+a_{2}+\ldots \ldots+a_{n-1}\right)\left(W_{n-1}-T_{n-1}\right)(i+1)^{n} \tag{5}
\end{gather*}
$$

This can be written as:

$$
\begin{align*}
a_{1}(i+1)\left[\prod_{m=1}^{n-1}\left(p_{m}+1\right)(1+i)^{n}-i\right]= & a_{1}(i+1)+a_{1}\left(W_{1}-T_{1}\right)(i+1)^{2}+a_{1}\left(p_{1}+1\right)\left(W_{2}-T_{2}\right)(i+1)^{3}+\ldots \ldots \ldots \ldots \\
& \ldots \ldots \ldots \ldots+a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right) \ldots\left(\mathrm{p}_{\mathrm{n}-2}+1\right)\left(W_{n-1}-T_{n-1}\right)(i+1)^{n} \tag{6}
\end{align*}
$$

Where:

$$
W_{n}=P_{n}+1, \mathrm{~T}_{\mathrm{n}}=1-\left(P_{n}+1\right)(i) \text { for }\left(\mathrm{P}_{\mathrm{n}}+1\right)(\mathrm{i}) \text { between } 0 \text { and } 1
$$

(that is, $\left.0<\left(P_{n}+1\right)(i)<1\right) ; P_{n}=$ performance rate i.e. such that $p_{n+1} \leq$ or $\geq \quad p_{n}$ and $\mathrm{T}_{\mathrm{n}}=$ is the firm performance other than
bank's deposit interest. The deposit interest rate should not be included into the project performance since it is always less than the bank lending interest rate; $\mathrm{i}=$ bank interest rate which is

Table 1. Present value of EVA Birr in millions.

| Variable | $\begin{gathered} \text { Base } \\ (\text { Year }=0) \end{gathered}$ | $\begin{gathered} 2003 \\ (\text { Year }=1) \end{gathered}$ | $\begin{gathered} 2004 \\ (\text { Year }=3) \end{gathered}$ | $\begin{gathered} 2005 \\ (\text { Year }=4) \end{gathered}$ | $\begin{gathered} 2006 \\ (\text { Year }=5) \end{gathered}$ | $\begin{gathered} 2007 \\ (\text { Year }=6) \end{gathered}$ | $\begin{gathered} 2008 \\ (Y e a r=7) \end{gathered}$ | $\begin{gathered} 2009 \\ (\text { Yea } r=8) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Net operating profit after tax | 12 | 15 | 17 | 19 | 63 | 71 | 81 | 74 |
| Less: Cost of capital | 1 | 2 | 3 | 3 | 3 | 5 | 7 | 8 |
| Economic value added | 11 | 13 | 14 | 16 | 60 | 66 | 74 | 66 |
| Performance rate |  | 0.18 | 0.08 | 0.14 | 2.75 | 0.10 | 0.12 | -0.11 |
| PV of EVA | 10 | 11.6364 | 0.8463 | 1.5887 | 35.1784 | 4.4772 | 5.8611 | -5.9588 |

between 0 and 1; $\mathrm{n}=$ investing period; $W_{n-1}-T_{n-1}=$ A portion of performance rate by which the excess consumption should be saved into a bank account.

The future value formula calculates each periods cash flow whose magnitude move along with the performance rate of the business which can enable the person to save a minimum of his / her performance in specified period.

## DISCUSSION AND CONCLUSION

The paper deeply focuses on how business firms determine their cash inflow or out flow based on their economic profit (or economic value added). As revealed by the formulae, the first payment is excess above consumption, which can be assigned for payment of debt or for saving into bank account and is a very small amount. The next after the first cash flow amounts progress or growth along with the business firm performance rate. Since this performance rate shows the relative level of growth of one's firm, current to last economic profit, it embraces all activities of the firm.

EVA reflects net of the cash generated and the cash invested by the business firm. As EVA fluctuates from period to period, the net cash left to the business firm also fluctuates from period to period. This fluctuation of the firm's cash inflow or out flow can exactly be reflected by the business performance rate $\left(P_{n}\right)$.
Financing organs, like banks, use ordinary annuity formula so as to determine loan capacity as well as loan repayment of the borrowers. Since the formula doesn't contain any measure of the performance of the borrowing organ, Most of the business loan are seen getting into nonperforming loan category, letting other things being constant. Because of this, banks always lay their own rules and regulations so as to minimize nonperforming loans at hand but couldn't reach at a conclusive solution for long period of time. Unless otherwise, there exist stiff control to collect the disbursed loan, the increment of bad debt from period to period will exactly harm the economic condition of the country as a whole. However, the present value formula stated by this paper calculates the projected fluctuating repayment amount along with the performance of the borrowing organ based on the real cash on hand which is excess above the consumption.

The future value formula enable people, who have no excess cash at hand for investment, to decide now on saving a portion of income according to their earnings growth from period to period so as to realize their dream after some fixed period. This also encourages the saving habits of people who have low income and those who are salaried, people who get their income from their employment at a fixed period interval such as monthly or annually.
Most business organ uses EVA as a measure of both value and performance. In Ethiopia, huge business firms like construction and business bank sc uses EVA as a measure of its performance. Construction and business bank sc is the only bank long stayed leading the market by lending long term construction loan in Ethiopia (Construction and Business Bank SC: www.cbb.com.et). Table 1 shows real past data of the bank for illustration using formulae of this paper in value creation.
As we have discussed, performance rate of each year is calculated on the basis of the following formula:

$$
P_{n}=\frac{E V A_{n+1}-E V A_{n}}{E V A_{n}}
$$

Such that:

$$
\begin{aligned}
& P_{1}=\frac{(13-11)}{11}=0.18 \quad P_{2}=\frac{(14-13)}{13}=0.08 \quad P_{3}=\frac{(16-14)}{14}=0.14 \\
& P_{4}=\frac{(60-16)}{16}=2.75 \quad P_{5}=\frac{(66-60)}{60}=0.10 \quad P_{6}=\frac{(74-66)}{66}=0.12 \\
& P_{7}=\frac{(66-74)}{74}=-0.11
\end{aligned}
$$

Assume that $E V A_{0}=a_{1}$ is the base year economic value added which can be assumed as an initial amount, interest rate (i) $=10 \%$, present value of each year EVA (Economic value added) is calculated as follow:

First year PV of EVA $=\frac{a_{1}}{(i+1)^{1}}=\frac{11}{(0.10+1)^{1}}=10$

Second year PV of EVA =

$$
\frac{a_{1}\left(p_{1}+1+i\right)}{(i+1)^{2}}=\frac{11(0.18+1+0.10)}{(0.10+1)^{2}}=\frac{14.08}{(1.1)^{2}}=11.6364
$$

Third year PV of EVA = $\frac{a_{1}\left(p_{1}+1+i\right) p_{2}}{(i+1)^{3}}=\frac{11(0.18+1+0.10)(0.08)}{(0.10+1)^{3}}=\frac{1.1264}{(1.1)^{3}}=0.8463$

Fourth year PV of EVA $=\frac{a_{1}\left(p_{1}+1+i\right)\left(p_{2}+1+i\right) p_{3}}{(i+1)^{4}}=\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14)}{(0.10+1)^{4}}=\frac{2.3260}{(1.1)^{4}}=1.5887$

Fifth year PV of EVA =

$$
\begin{aligned}
& \frac{a_{1}\left(p_{1}+1+i\right)\left(p_{2}+1+i\right)\left(p_{3}+1+i\right) p_{4}}{(i+1)^{5}} \\
& =\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10) 2.75}{(0.10+1)^{5}} \\
& =\frac{56.6551}{(1.1)^{5}}=35.1784
\end{aligned}
$$

Sixth year PV of EVA =

$$
\begin{aligned}
& \frac{a_{1}\left(p_{1}+1+i\right)\left(p_{2}+1+i\right)\left(p_{3}+1+i\right)\left(p_{4}+1+i\right) p_{5}}{(i+1)^{6}} \\
& \frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10)}{(0.10+1)^{6}} \\
& =\frac{7.9317}{(1.1)^{6}}=4.4772
\end{aligned}
$$

Seventh year PV of EVA =

$$
\begin{aligned}
& \frac{a_{1}\left(p_{1}+1+i\right)\left(p_{2}+1+i\right)\left(p_{3}+1+i\right)\left(p_{4}+1+i\right)\left(p_{5}+1+i\right) p_{6}}{(i+1)^{7}} \\
& =\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10+1+0.10) 0.12}{(0.10+1)^{7}} \\
& =\frac{11.4217}{(1.1)^{7}}=5.8611
\end{aligned}
$$

Eighth year PV of EVA =

$$
\begin{aligned}
& \frac{a_{1}\left(p_{1}+1+i\right)\left(p_{2}+1+i\right)\left(p_{3}+1+i\right)\left(p_{4}+1+i\right)\left(p_{5}+1+i\right)\left(p_{6}+1+i\right) p_{7}}{(i+1)^{8}} \\
& =\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10+1+0.10)(0.12+1+0.10)(-0.11)}{(0.10+1)^{8}} \\
& =\frac{-12.7732}{(1.1)^{8}}=-5.9588
\end{aligned}
$$

Table 2. Deposit.

| Variable | Base $(\text { Year }=0)$ | $\begin{gathered} 2003 \\ (\text { Year }=1) \end{gathered}$ | $\begin{gathered} 2004 \\ (\text { Year = 2) } \end{gathered}$ | $\begin{gathered} 2005 \\ (\text { Year }=3) \end{gathered}$ | $\begin{gathered} 2006 \\ (\text { Year }=4) \end{gathered}$ | $\begin{gathered} 2007 \\ (\text { Year = 5) } \end{gathered}$ | $\begin{gathered} 2008 \\ (\text { Year }=6) \end{gathered}$ | $\begin{gathered} 2009 \\ (\text { Year }=7) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Performance rate $\left(p_{n}\right)$ |  | 0.18 | 0.08 | 0.14 | 2.75 | 0.10 | 0.12 | -0.11 |
| $W_{n}=p_{n}+1$ |  | 1.18 | 1.08 | 1.14 | 3.75 | 1.10 | 1.12 | 0.89 |
| $T_{n}=1-\left(p_{n}+1\right) i$ |  | 0.882 | 0.892 | 0.886 | 0.625 | 0.89 | 0.888 | 0.911 |
| $W_{n}-T_{n}$ |  | 0.298 | 0.188 | 0.254 | 3.125 | 0.21 | 0.232 | -0.021 |

From Table 1, initial amount $\left(E V A_{0}\right)=11$; deposit Interest rate $(i)=10 \%$.

The total value of each periods EVA's present values is easily sumed by the formula as follow:

Present value of EVAs =
$10+11.6364+0.8463+1.5887+35.1784+4.4772+5.8611+$ $(5.9588)=63.6293$

The accuracy of this result can be checked by the following present value formula:

$$
\left[\prod_{m=1}^{n-1}\left(P_{m}+1+i\right)+(i+1)^{n-1}\right]\left(\frac{a_{1}}{(1+i)^{n}}\right)=
$$

$[(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)$ $(0.10+1+0.10)(0.12+1+0.10)(-0.11+1+0.10)+$

$$
\left.(0.10+1)^{(8-1)}\right]\left[\frac{11}{(0.10+1)^{8}}\right]=63.6293 \text { Ethiopian dollars, }
$$

If the bank had had projected a cash inflow or out flow standing on a base year, the value of those cash within the given life time would have been the afore-calculated result. This helps the bank to decide comparing with the initial investment as to whether the investment is feasible or not.
Each of the afore calculated PV of EVA is positive except the last term, of year 2009, by which the bank shows lowest performance relative to the year 2008 performance. Positive EVA increases the value of the firm whereas the negative EVA reduces the value of the firm. As per this paper recommendation, the bank should have shown a strong performance for which the current performance of the bank should be greater than the last otherwise the formulae detect as the firm has not been performed well in that given period.

The main objective of a bank is accepting deposit from the society and lending the collected deposit amount to the people who meet liquidity shortage. The loan officer of the bank ought to provide a strong realistic feasibility study regarding the project so as to identify the eligibility of the person to be a borrower based on the projected net cash inflow of the business. Why the main problem
stayed for long is that projected cash flow of the business ignores future risk of interest rate fluctuation and future inflation. Because of these short coming, the loan repayment set by the bank disregarding inflation and risk in interest rate fluctuation will become a burden to the borrower when this factors happens in the real market. As a result of this, the borrowers fail to climb their loan repayment obligation within the given loan life and this causes the bank to fail in collecting the disbursed loan amount properly. The increase of uncollectable loans from period to period leads the bank to meet liquidity problem which will also levy a shadow to the country economy as a whole. However, the bank liquidity problem will have a solution if the bank has a habit of keeping reserve a portion of net cash on hand so as to fill back the shortage of liquidity.
The future value formula of this paper helps the bank in decision making of holding reserve basing the real cash on hand for which a small portion of it can be considered as initial deposit amount. This small initial amount will progress according to the performance of the bank EVA. Assuming again, construction and business bank sc as a principal thinker of holding liquidity reserve, and standing on the base year if it had had projected its cash reserve deposit by the end of year 2009, it would have had a deposit amount as shown in Table 2.
Accordingly, the calculation of the current cash flow to get the future value up to year 2008 is shown in Table 3. All the current cash investments of each period are positive in magnitude except the last, year 2009, cash investment.
The negative sign indicates that the bank performance in the period is less than the last year so the bank ought to perform currently more than the last period.
As have been seen, the steps to arrive at a conclusion is very tedious and time consuming, However, the future value formula of this paper calculates the above cash flow easily, as follows:
$a_{1}(i+1)\left[\prod_{m=1}^{n-1}\left(p_{m}+1\right)(1+i)^{n}-i\right]$

Table 3. Calculation of deposit cash flow.

| Year | Cash flow |
| :---: | :---: |
| 1 | $a_{1}(i+1)=11(1.10)=12.10$ |
| 2 | $a_{1}\left(W_{1}-T_{1}\right)(i+1)^{2}=11(0.298)(1.10)^{2}=3.9664$ |
| 3 | $a_{1}\left(p_{1}+1\right)\left(W_{2}-T_{2}\right)(i+1)^{3}=11(1.18)(0.188)(1.10)^{3}=3.2480$ |
| 4 | $a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right)\left(W_{3}-T_{3}\right)(i+1)^{4}=11(1.18)(1.08)(0.254)(1.10)^{4}=5.2132$ |
| 5 | $\begin{aligned} & a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right)\left(p_{3}+1\right)\left(W_{4}-T_{4}\right)(i+1)^{5} \\ & =11(1.18)(1.08)(1.14)(3.125)(1.10)^{5}=80.4297 \end{aligned}$ |
| 6 | $\begin{aligned} & a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right)\left(p_{3}+1\right)\left(p_{4}+1\right)\left(W_{5}-T_{5}\right)(i+1)^{6} \\ & =11(1.18)(1.08)(1.14)(3.75)(0.21)(1.10)^{6}=22.2951 \end{aligned}$ |
| 7 | $\begin{aligned} & a_{1}\left(p_{1}+1\right)\left(p_{2}+1\right)\left(p_{3}+1\right)\left(p_{4}+1\right)\left(p_{5}+1\right)\left(W_{6}-T_{6}\right)(i+1)^{7} \\ & =11(1.18)(1.08)(1.14)(3.75)(1.10)(0.232)(1.10)^{7}=29.8033 \end{aligned}$ |

Summing the current cash investment, we have the future value as:
$12.10+3.9664+3.2480+5.2132+80.4297+22.2951+29.8033=157.0557$.
$=11(1.10)\left[(1.18)(1.08)(1.14)(3.75)(1.10)(1.12)\left(1.10^{7}\right)-\right.$ $(0.10)]=157.0557$.

Generally, the formulae can easily help creditors, investors and persons who have low income in order to facilitate their decision shortly and accurately. They can also help to estimate either the present or the future cash flow of a firm more perfectly.

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