Full Length Research Paper

Optimal pricing for internet service providers: Fuzzy geometric programming model

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An internet service provider (ISPs) normally purchases a bandwidth from a vendor and allocates it to different services. The primary concern is to maintain a quality service and, at the same time, maximize the profitability. This paper presents a mathematical model to find the optimal pricing strategy for an internet service provider when the primary concern on many pricing strategies is the absence of historical data for services which makes it difficult to provide a good estimate on the necessary pricing information such as price elasticity and the parameters are taken as a triangular fuzzy numbers. This paper presents a mathematical model to find the optimal pricing strategy in the fuzzy environment. The resulted problem is formulated as a general form of possibilistic geometric programming (PGP). The implementation of the proposed method is demonstrated via a case study on an ISP in Iran.

Key words: Revenue management, pricing, geometric programming, fuzzy programming, internet service provider.

INTRODUCTION

Revenue Management (RM) has considerable potential for economical production in order to maximize the total profit. It can significantly help organizations keep their advantages in competitive markets. Belobaba, (1987), McGill and Ryzin (1999) discussed about the origin of the revenue management. McGill and Ryzin (1999) catego-rized revenue management problems into forecasting, overbooking, pricing and capacity control. Chiang et al. (2007) extended their model by considering additional assumptions. As McGill and Ryzin (1999) mentioned, pricing is one of the most important aspects of RM. The optimal pricing has been investigated by many resear-chers during the past few decades. In the proposed models, some factors such as selling price, customer segmentation, marketing expenditure, etc have been frequently considered as the most effective factors in the optimal pricing problem. Burger and Fuchs (2005) argued about the effects of dynamic pricing on RM and showed

that it can lead to a decrease in complexity of pricing process. Frank (2003) discussed about application of six-sigma at pricing. Lee and Kim (1993) are the first people who proposed an integrated marketing and production models. They considered demand as an exponential function of price and marketing expenditures and, at the same time, the cost of production was also considered as a function of demand. Therefore, the resulted problem was formulated as a geometric programming model. In another attempt, Kim and Lee (1998) extended their model when the capacity of a market was also incorpora-ted either fixed or variable. Sadjadi et al. (2005) deve-loped Lee´s model under some additional assumptions. In other research, Sadjadi et al. (2010) presented a new marketing and pricing model in the fully uncertain envi-ronment where decision variables were fuzzy numbers. Also, they developed a decision support system to inference vector of decision variables without solving geometric programming problem directly. Tsaia and Hung (2007) proposed an integrated real options (IRO) approach for solving competitive/dynamic pricing and RM problem in internet retailing where environment was uncertain. They used analytic hierarchy process (AHP) to

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determine the weights of the auction commodities. Nair and Bapna (2001) studied the internet service provider (ISP) market structure and explained that demand and supply in this sector is different from other industry sectors, so that demand and service happened at the same time. In other words, the clients, at any time of day, may require the services and therefore, every moment system must be ready to serve and the possibility of overbooking does not exist.

Note that we have a high level of competition on ISP market on one side and technology is under significant changes on a daily basis and there is a need to develop a good strategy for assigning the suitable price for ISP services.

In this paper, a mathematical model is developed for ADSL service pricing. Since most of the real world problems occurred in the uncertain environment, to tackle this uncertainty, the proposed model is formulated in the fuzzy environment as possibilitic geometric programming model. To show the performance of the presented model, details of implementation of proposed method on one of ISP´s located in Iran will be explained.

PROBLEM STATEMENT

A fuzzy pricing model for ISP services is developed under the following assumptions and notations:

Notations

α_j: Selling price elasticity to demand

γ_j : Type of service (kb/second)

β_j : Share ratio

P_j : Selling price (decision variable)

D_j: Services demand

B: Total bandwidth that company receives per period

k_j: Predetermined constant

Assumptions

I: Selling price elasticity to demand is uncertain and depicted in the form of triangular fuzzy number $\tilde{\alpha} = (\alpha_l, \alpha_m, \alpha_r)$.

II: $\tilde{k}$ is uncertain and depicted in the form of triangular fuzzy number $\tilde{k} = (k_l, k_m, k_r)$

III: Price is a function of demand so that it has inverse relation with the selling price as follows:

$$D = \tilde{k} P^{-\tilde{\alpha}}. \tag{1}$$

Based on the notations and assumptions expressed, the

pricing for ISP services is developed in fuzzy environment.

Fuzzy pricing model

In this section, the pricing model for ISP services is developed in the fuzzy environment. As mentioned before, the objective function is the maximization of revenue and there is a constraint on bandwidth. If there are in different services, mathematical pricing model can be formulated as follow,

$$\max \sum_{j=1}^{n} P_j D_j \tag{2}$$

subject to

$$\sum_{j=1}^{n} \frac{\gamma_j D_j}{\beta_j} \leq B$$

Replacing D in Model (2) by (1) yields:

$$\max \sum_{j=1}^{n} \tilde{k}_j P_j^{1-\tilde{\alpha}_j} \tag{3}$$

subject to

$$\sum_{j=1}^{n} \frac{k_j \gamma_j P_j^{-\tilde{\alpha}_j}}{\beta_j} \leq B$$

Model (3) is a possibilitic geometric programming that $\hat{\alpha}$ and $\tilde{k}$ have triangular possibility distribution. In order to solve Model (3), fuzzy goal programming method is used. Following goal programming concept, an aspiration level such as $Z = (Z_l, Z_m, Z_r)$ is chosen and the objective function is added to constraint via $\tilde{Z}$ as follows:

$$\max \tilde{Z} \tag{4}$$

subject to

$$\sum_{j=1}^{n} \tilde{k}_j P_j^{1-\tilde{\alpha}_j} \geq (Z_l, Z_m, Z_r)$$

$$\sum_{j=1}^{n} \frac{\tilde{k}_j \gamma_j P_j^{-\tilde{\alpha}_j}}{\beta_j} \leq B.$$

Model (4) is a possibilitic geometric programming in which $\tilde{\alpha}_j$ and $\tilde{k}_j$ have triangular possibility distributions.

After defuzzification, Model (4) is reformulated as Model (5):

$$\max \sqrt[3]{Z_l.Z_m.Z_r} \qquad (5)$$

subject to

$$\sum_{j=1}^{n} k_j^l P_j^{(1-\alpha_j^r)} \geq Z_l,$$

$$\sum_{j=1}^{n} k_j^m P_j^{(1-\alpha_j^m)} \geq Z_m,$$

$$\sum_{j=1}^{n} k_j^r P_j^{(1-\alpha_j^r)} \geq Z_r,$$

$$\sum_{j=1}^{n} \frac{k_j \gamma_j P_j^{-\tilde{\alpha}_j}}{\beta_j} \leq B.$$

As it is shown in Model (5), geometric mean is used for objective function defuzzification and the first constraint of Model (4) is defuzzified using strong comparison rule (SCR) method. The first, the second and the third constraints in (5) are not in posynomial form. To solve this problem we use a simple lemma from Beightler and Phillips (1976). To this end, consider the first constraint of the Model (5) that can be rewritten as follows:

$$Z_1^{-1} k_1^l P_1^{(1-\alpha_1^r)} + Z_1^{-1} k_2^l P_2^{(1-\alpha_2^r)} + \cdots + Z_1^{-1} k_n^l P_n^{(1-\alpha_n^r)} \geq 1 \quad (6)$$

Parameter t_{1j} is defined as follow:

$$t_{1j} = \frac{k_j^l P_j^{(1-\alpha_j^r)}}{\displaystyle\sum_{j=1}^{n} k_j^l P_j^{(1-\alpha_j^r)}}, \qquad j=1,\cdots,n \qquad (7)$$

where

$$\sum_{j=1}^{n} t_{1j} = 1.$$

So the first constraint in (5) can be rewritten as follow:

$$\prod_{j=1}^{n} \left(\frac{k_j^l P_j^{(1-\alpha_j^r)} Z_l^{-1}}{t_{1j}} \right)^{-t_{1j}} \leq 1 \qquad (8)$$

Similarly, the define variables t_{2j} and t_{3j} as follow:

$$t_{2j} = \frac{k_j^m P_j^{(1-\alpha_j^m)}}{\displaystyle\sum_{j=1}^{n} k_j^m P_j^{(1-\alpha_j^m)}}, \qquad j=1,\cdots,n \qquad (9)$$

$$t_{3j} = \frac{k_j^r P_j^{(1-\alpha_j^r)}}{\displaystyle\sum_{j=1}^{n} k_j^r P_j^{(1-\alpha_j^r)}}. \qquad j=1,\cdots,n \qquad (10)$$

As well as first constraint, the second and the third constraints can be reformulated as follows:

$$\prod_{j=1}^{n} \left(\frac{k_j^m P_j^{(1-\alpha_j^m)} Z_m^{-1}}{t_{2j}} \right)^{-t_{2j}} \leq 1, \qquad (11)$$

$$\prod_{j=1}^{n} \left(\frac{k_j^r P_j^{(1-\alpha_j^r)} Z_r^{-1}}{t_{3j}} \right)^{-t_{3j}} \leq 1, \qquad (12)$$

So, Model (5) can be rewritten as follow:

$$\max \sqrt[3]{Z_l.Z_m.Z_r}$$

subject to

$$\prod_{j=1}^{n} \left(\frac{k_j^l P_j^{(1-\alpha_j^r)} Z_l^{-1}}{t_{1j}} \right)^{-t_{1j}} \leq 1$$

$$\prod_{j=1}^{n} \left(\frac{k_j^m P_j^{(1-\alpha_j^m)} Z_m^{-1}}{t_{2j}} \right)^{-t_{2j}} \leq 1,$$

$$\prod_{j=1}^{n} \left(\frac{k_j^r P_j^{(1-\alpha_j^r)} Z_r^{-1}}{t_{3j}} \right)^{-t_{3j}} \leq 1,$$

$$\sum_{j=1}^{n} \frac{k_j^r \gamma_j P_j^{-\alpha_j^l}}{\beta_j} \leq B. \qquad (13)$$

To defuzzify fourth constraint of Model (5), SCR method is also used. Since the right hand side of the fourth constraint of Model (5) is crisp, defuzzification yields one constraint as fourth restriction of Model (13).

Table 1. The results of the changes on different price elasticity items.

	α	p_1	p_2	p_3	Z_1	Z_2	Z_3	Z
α1 +20%	(4.56 ,5.04 ,5.4)	5007	68030	118187	9860280	30225308	105743910	31586763
α1 +20%	(3.04,3.36, 3.6)	84652	68030	118187	9860280	30225308	105743910	31586763
α2+20%	(3.24,3.36,3.48)	19734	32688	57675	8119298	14976695	46267491.98	17785666
α2-20%	(2.16,2.24,2.32)	24898	981295	1893039	123363302	446238559	1570035471	442134273
α3+20%	(2.52, 2.64, 2.7)	23885	62173	117153	7818541	28266259	99467126	28012887
α3-20%	(1.68,1.76,1.8)	23020	448930	885278	125978910	230436379	708270551	273957183

Note that Problem (8) is a parametric optimization problem. The regular procedure to solve (13) is to solve it different times. Each time, we estimate t_{ij} ; $i = 1,2,3$; $j = 1,2,...,n$ using (7), (9) and (10) and the process continues until there are practically no changes on parameters. The experience shows that we need only a few iterations to reach the optimal solution.

The study presents the implementation of the developed model on one of ISPs located in IRAN that serves ADSL services.

CASE STUDY

The study implemented the proposed model of this paper for one of the ISPs located in Iran which offers ADSL services. The company offered three types of desired services: 128, 256, 512 kb / s.

Based on the experience of experts parameters k_j ; $j = 1,2,3$ and α_j ; $j = 1,2,3$ are estimated as follows:

$$\alpha_1 = [3.8, 4.2 , 4.5],$$
$$\alpha_2 = [2.7 , 2.8 , 2.9],$$
$$\alpha_3 = [2.1 , 2.2 , 2.25]$$

$$k_1 = [1.4 \times 10^7, 1.8 \times 10^7, 2.2 \times 10^7],$$
$$k_2 = [1.0 \times 10^7, 1.2 \times 10^7, 1.4 \times 10^7],$$
$$k_3 = [7.2 \times 10^{12}, 7.6 \times 10^{12}, 7.8 \times 10^{12}]$$

Other parameters are:
$$b_1 = b_2 = b_3 = 8$$
$$B = 51200 \text{ kb/sec}$$
$$\gamma_1 = 128 \text{ kb/sec}$$
$$\gamma_2 = 256 \text{ kb/sec}$$
$$\gamma_3 = 512 \text{ kb/sec}$$

Problem is solved using CVX software and the optimal results are obtained as follow:

$$P_1 = 2.3864 \times 10^4, P_2 = 6.8030 \times 10^4, P_3 = 1.1819 \times 10^5$$

$$\tilde{Z} = (9.8603 * 10^6 , 3.0225 * 10^7, 1.0574 * 10^8)$$

Sensitivity analysis

The changes on price elasticity of three products by decrease/increase in their values in a range of 20% were studied. Table 1 summarizes the experimental results. Also, the effects on changes in price elasticity on prices were studied. As can be observed from Table 1 the change on α_1 only changes on the first service and it has no significant impact on the total revenue. Note that k_1 has relatively lower value compared with other ones and this is another reason to believe that any change on α_1 does not change the overall revenue. In this case study, the change on the price of the first product does not make a big change on total revenue. On the other hand, based on the results of table, when the price of the first product increases the prices of the other two products remain with no changes. In fact, in some cases the price of the second service is higher than the first one. In such a case, customer may prefer to choose the second service due to higher internet speed and better availability. The increase on α2 has much more influence on prices since it has higher value of k2. As can be observed from the table, a 20% change on the price of the second service reduces the price of the second and the third products, significantly. Since the second product is more popular among people where the case study is implemented, it can be concluded that this product has a higher competitive advantage against the other two products. The changes on the third product are not as significant as the impacts of price changes of the second product. In summary, it could be concluded that the second product plays an important role on the market.

CONCLUSION

In this paper, a mathematical model to formulate and solve pricing Internet Service Providers (ISP) problems

was proposed. Developing model in the fuzzy environment increases model's stability against different real world situations. Whereas in the real world problems, most of effective parameters are estimated by experts as linguistic variables and on the other hand, linguistic terms usually quantified using fuzzy numbers, in the proposed model, uncertain parameters k_j:$j=1,2,3$ and α_j; $j=1,2,3$ are considered as fuzzy numbers with triangular possibility distribution. Finally, model is formulated as a possibilitic geometric programming and the effectiveness of the model has been illustrated via one of the ISP's located in Iran.

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