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*Review*

# Oscillation Theory of International Economic Integration

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**An article provides a new insight to the modelling of the dynamics of international economic integration using qualitative properties of an oscillation theory. Proposed method of computing the GDP after economic integration enables to compute the benefits of economic integration for member states on the union: one needs to know the rates of domestic savings, amplitudes and frequencies of the cycles of the value added of sectors for united countries / regions. A new vision of the dynamics of economic integration proposed in the article is similar to unification of water basins previously separated by sluices, in which the waves of the value added generated by economic entities, interact with each other. It was analytically proved that economic integration causes growth of the GDP of unified states.**

**Keywords:** The dynamics of international economic integration; oscillation theory; value added of sectors; GDP; domestic savings rate

## INTRODUCTION

International economic integration is a process of a formal unification of previously separate economic areas: after canceling tariff and non-tariff barriers it increases volume of trade among members of economic union, generates more economic activity and thus changes inner content of integrated economies towards better welfare.

Ability to estimate future benefit of economic integration of the potential blocs and groupings has always been of practical importance. For instance, there is a long-planned FTAA project of Pan-American free trade area, with mainly political obstacles staying against its implementtation. Computation of the benefits for the countries involved may stimulate its evolution. The other example may be the Shanghai Organization of Cooperation, currently consisting of China, Russia and Central Asian states, which is not an integrated grouping yet: finding potential benefits here may stimulate steps towards their economic integration too. Such intra- and inter-continental cooperation among different states and groupings worldwide certainly needs an estimation of integration's real value especially since globalization now became a factor permanently influencing national economic policies. Theory of international economic integration was originally based on Viner's idea (1950) which considers static case of two integrating states whose trade flows efficiency is compared with the one for the rest of the world via trade creation and trade diversion effects. After nearly six decades of development, the theory of economic integration stays far behind economic practice (El-Agraa, 1998; Balassa, 1967; Cooper and Massel, 1965; Lipsey, 1957; Meade, 1956; Viner, 1950; Devlin and French-Davis, 1998; Jovanovich, 1998; Tovias, 1994; Robson, 1998). Most experts note that the modelling economic integration lacks the models describing the dynamics of the phenomenon (Winters, 1997; Ruiz Estrada, 2004).

One of the current approaches to the modelling economic integration is called Regional Integration Evaluation Methodology studying different areas of development involving political, social, economic and technological analysis. Its idea is to demonstrate that the regional development can affect the evolution of regional integration process based on the application of a group of indexes and graphs. The group of indexes and graphs show the evolution and stages of the regional integration process of region from a multi-dimensional analysis (Ruiz Estrada, 2004). The General Equilibrium methodology helped to develop Gravity Model, Import-Growth Simulation and several other regression models (Mordechai and Plummer, 2002).

Another method in the modelling is the Global Dimension of Regional Integration Model (GDRI-Model) used as a measuring tool for studying regional integration, apply-

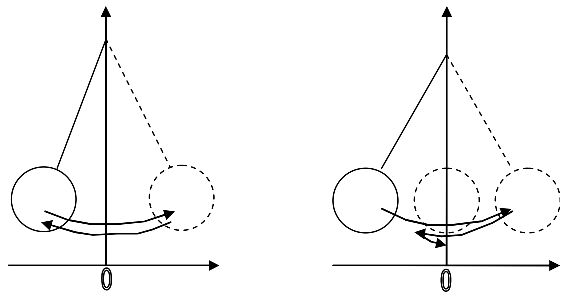



**Figure 1.** Oscillations in case of no-friction and friction.

ing dynamic and general equilibrium analysis to show the past and present situations in the regional integration process of any region based on a set of indexes and graphs. Its field application is not constrained by region or the development stage of each member interested in integrating into a single regional bloc. The GDRI-Model is not used for forecasting, mainly focusing on the past and present situations in the regional integration process (Ruiz Estrada, 2004).

Economic integration assumes merging of at least two separate different economies, with the process itself essentially being dynamic one and hence needing non-static tools applied to the modeling of economic integration. Recently there appeared several articles (Bergin and Glick, 2007; Landesmann and Stehrer, 2002; Landesmann and Stehrer, 2006; Tulpule, 2002) in this direction both revisiting the issue of modeling economic integration as well as making overviews and generalizations in its dynamic modeling. At the same time, one has to admit, many economists and politicians in their daily practice seeking to find out specific benefit at forming an economic union still cannot do it for very reason that they are unable to implement wide statistic data they posses due to a lack of suitable economic model.

Present article may help to overcome existing difficulties in methodology of economic integration theory based on oscillation theory approach. Its main advantage is in the fact that one may use magnitudes taken from real economy, such as domestic savings rates and values added of sectors, and make an estimate of potential benefit for states in case of their possible unification. Ideas and concepts proposed in the article include:

i) Definition of the GDP as a difference between sector's value added and respective investment.
ii) Consideration of the dynamics of the value added using its oscillating origin.
iii) A formula allowing computation of the GDP after economic integration.
iv) Analytical proof that economic integration causes growth of the GDP of unified regions.
v) A new vision of the dynamics of economic integration, similar to unification of water basins previously separated by sluices, in which the waves of the value added generated by economic entities interact with each other.

Further sections of the article provide a brief overview of pendulum's simplest oscillations (section II) and the modeling of regional economic dynamics and economic integration (section III). Section IV provides modeling of economic integration for 2004 EU expansion, with results of simulations presented in the last section V.

## Oscillation theory

The very basics: It considers (Andronov et al., 1949) dynamics of one pendulum with no friction, with the friction (Figure 1) and enforced oscillations. One with no friction, or the so-called "ideal pendulum", for *y* as a variable is described by the following equation:

$$\ddot{y} + \omega^2 y = 0, \qquad (1)$$

Where $\omega = \frac{2\pi}{T}$ and $T$ stand for natural frequency and period of oscillations. Friction, present in the real world, adds a little change to our initial equation (1):

$$\ddot{y} + 2j\dot{y} + \omega^2 y = 0 \qquad (2)$$

Where $j$ - damping factor representing friction. When oscillations of *y* die out, one may swing them again by applying external force *f*: it is in the same way how, for instance, we restart clocks, thus inputting there a human's force. Equation (2) now takes the following form:

$$\ddot{y} + 2j\dot{y} + \omega^2 y = f \qquad (3)$$

Solution of (3) has an interesting important feature: if *f* is periodic function of a kind such as $f = A \sin \Omega t$, then *y* switches to the frequency of external force $\Omega$ and ceases to notice external influence. In other words, dynamics of the system in (3) becomes the one like in Equation (2),

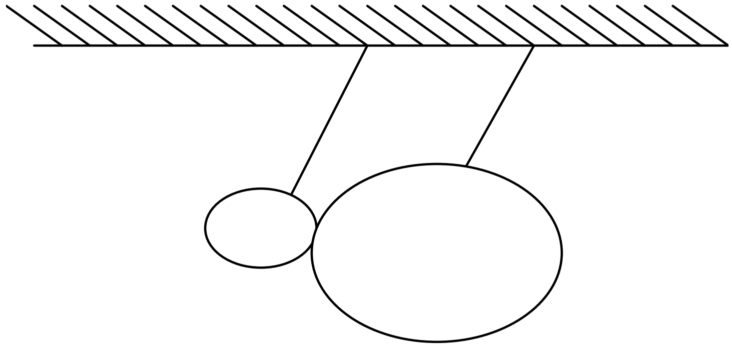


**Figure 2.** Transition to the frequency of external influence: hanging balls.

with new natural frequency $\Omega$ . Indeed, if we imagine hanging ball which oscillates from one side to the other, then by applying a force of another ball, much bigger than the first one, the small ball in just a while will oscillate with the frequency of its bigger counterpart. Actually, it usually looks like the smaller ball is "glued" to bigger one (Figure 2).

Modelling regional economic development and economic integration: It is commonly recognized that one of the main goals of economic integration is to raise the welfare of integrated states with expected rate of growth higher than in case when countries would not integrate; otherwise there is no reason for economic unification.

The welfare may be characterized by magnitude of gross domestic product y(x, y, t) per capita. Taken together, we may state that finding the terms of the GDP growth for the united areas, or $\dot{Y} \equiv \frac{dY}{dt} > 0$ during economic integration, is equivalent to the analysis of the dynamics of economic integration.

We start a modeling by deducting the differential equation responsible for the dynamics of regional GDP. Then we will merge regions with attempt to make qualitative analysis of the GDP dynamics due to economic integration. General terms of the modeling the GDP dynamics derive from the point that prior to economic integration in the moment of time $t_0$ we had dynamics of the GDP for regions I and II as a sum of dynamics of the revenue $Y_i$ for *n* and *m* sectors of respective regional economies:

$$Y_I = Y_1 + \ldots + Y_n ; \; Y_{II} = Y_{n+1} + \ldots + Y_{n+m} ; \tag{4}$$

$$\dot{Y}_I(t_0) = \dot{Y}_1(t_0) + \ldots + \dot{Y}_n(t_0) ; \; \dot{Y}_{II}(t_0) = \dot{Y}_{n+1}(t_0) + \ldots + \dot{Y}_{n+m}(t_0) \tag{5}$$

$$\dot{Y}_I(t_0) > 0; \; \dot{Y}_{II}(t_0) > 0. \tag{6}$$

Conditions (4)-(6) seem obvious and stand for requirement of economic growth in respective regions. After economic integration, which takes place at the moment of time $t_1 \; (t_1 > t_0)$, the GDPs and their respective dynamics will merge:

$$Y = Y_I + Y_{II} = Y_1 + \ldots + Y_n + Y_{n+1} + \ldots + Y_{n+m} ; \tag{7}$$

$$\dot{Y} > 0; \tag{8}$$

$$\dot{Y}(t_1) > \dot{Y}(t_0) \text{, or} \tag{9}$$

$$\dot{Y}_1(t_1) + \ldots + \dot{Y}_n(t_1) + \dot{Y}_{n+1}(t_1) + \ldots + \dot{Y}_{n+m}(t_1) > \dot{Y}_1(t_0) + \ldots + \dot{Y}_n(t_0) + \dot{Y}_{n+1}(t_0) + \ldots + \dot{Y}_{n+m}(t_0) \quad (10) \tag{10}$$

Here the last two expressions (9)-(10) stand for requirement of "more" economic growth which should take place due to economic integration compared to a case of separate states without economic unification.

We may further simplify analysis by considering a case of economic integration between just two neighboring states, with each of them having only one industrial sec-tor. Historically international cooperation between two sectors was observed within the framework of European Coal and Steel Union, although historically there were more than 2 states involved. Currently we may witness such cooperation between some Arabian states exporting oil, serving as a main engine for their economies. The other country in such a case may be the one which will consume or process imported oil. In reality such industrial "partner" for the first state will certainly have in its economy more than one sector, but we may try to model a case of "2 states ÷ 2 sectors" and then to take into attention sectors neglected in the beginning.

**Regional economic development model**

Let us consider gross domestic product $Y(x, y, t)$ as a main variable in modeling, where $x$ and $y$ are spatial co-ordinates and $t$ is a time. We will attempt to find conditions for the GDP growth from solution of its differential equation which we have to deduct. We start by letting revenue of each sector in economy be equal to $Y_i$, which is aggregated to:

$$Y = \sum_{i=1}^{n} Y_i \tag{11}$$

If $I$ is total investment in economy, and $I_i$ is investment in sector $i$, with $i \in [1; n]$, we have total investment in economy equal to the sum of current costs in each sector:

$$\sum_{i=1}^{n} I_i = I \tag{12}$$

For the revenue of sector $i$ we have relationship natural in a business sense defined as difference between amount obtained after the sales of manufactured products (gross income, which is multiplication of price $P_i$ per output $Q_i$, or just a value added in services sector) and its costs in the form of expenditures $I_i$:

$$Y_i = P_i Q_i - I_i \tag{13}$$

Indeed, every business in the world defines its profitability using this very logic first by investing into business and then selling the goods. If the outcome of its business activity $Y_i$ is positive, it makes profit, and if negative – it "looses" thus requiring some action to change the trend. It is assumed that the GDP as parameter of a national profit may also be defined using such an approach. By entering (12), (13) in (11) we find that the GDP is equal to the amount of gross products as a multiplication of price per output of the goods produced subtracting investments spent on them, i.e. expenditures:

$$Y = \sum_{i=1}^{n} P_i Q_i - I \tag{14}$$

Expression (14) thus links macro- and microeconomic parameters such as the GDP and investment on the one hand, and prices and outputs of economic sectors on the other.

Let us deduct second equation for the GDP $Y$, so we will use it together with (14). In case of an open economy an equilibrium of savings $S=sY$ and investment $I$ is upset. We will look at case close to such equilibrium where we may consider investments as savings in the beginning of the period plus its change caused by respective change of the GDP during this period:

$$I = sY + \dot{Y},$$

Where $0 \le s \le 1$, $s$ = const

Then we obtain:

$$\dot{Y} = I - sY \tag{15}$$

Spatially non-homogeneous allocation of the GDP was examined in a model of inter-regional trade (Puu, 1997). It was accounted for by using laplasian $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, which comes due to interregional trade by adding net export $X_n$ as a balance between exports $X$ and imports $M$:

$$\dot{Y} = I - sY + X_n, \tag{16}$$

where

$$X_n = X - M = \mu \nabla^2 Y, \tag{17}$$

and $\mu > 0$. We obtain a change of net export in the same way as we did for the GDP:

$$\dot{X}_n = \mu \nabla^2 Y - X_n \tag{18}$$

Let's differentiate (16) with respect to time and substitute values of $I$, $X_n$ using equations (14), (16)–(18). This leads to the following:

$$\ddot{Y} = \dot{I} - s\dot{Y} - \dot{X}_n = \left(\sum_{i=1}^{n} (\dot{P}_i Q_i + P_i \dot{Q}_i) - \dot{Y}\right) - s\dot{Y} + (\mu \nabla^2 Y - X_n) =$$
$$= \sum_{i=1}^{n} (\dot{P}_i Q_i + P_i \dot{Q}_i) - (1+s)\dot{Y} + \mu \nabla^2 Y - (\dot{Y} + sY - I) =$$
$$= \sum_{i=1}^{n} (\dot{P}_i Q_i + P_i \dot{Q}_i) - (2+s)\dot{Y} + \mu \nabla^2 Y - sY + \left(\sum_{i=1}^{n} P_i Q_i - Y\right) =$$
$$= \sum (\dot{P}_i Q_i + P_i \dot{Q}_i + P_i Q_i) - (2+s)\dot{Y} - (1+s)Y + \mu \nabla^2 Y.$$

Thus, differential equation of a second degree for the GDP we were seeking is as follows:

$$\boxed{\ddot{Y} + (2+s)\dot{Y} + (1+s)Y - \mu \nabla^2 Y = \sum_{i=1}^{n} \left(P_i Q_i + \frac{\partial}{\partial t} P_i Q_i\right)} \tag{19}$$

In the left hand side of (19) we have an equation quite

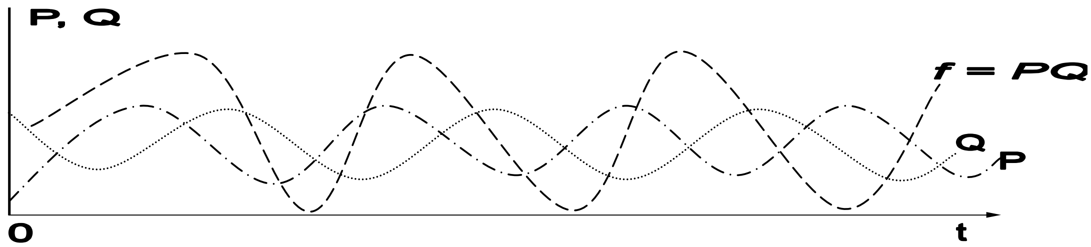



**Figure 3.** Cyclical oscillations of price, output and value added function.

similar to the one of enforced attenuated oscillations, where GDP $Y$ is a pendulum. As an extra element there one may see the term responsible for spatial dynamics of the GDP. Oscillations are enforced by terms at right which further will be called a force $f = \sum_{i=1}^{n}(P_i Q_i + \frac{\partial}{\partial t} P_i Q_i)$.

International economic integration model: "2 states $\div$ 2 sectors" case: From expression (19) one may see that dynamics of the GDP is influenced by respective dynamics of the value added and its temporal change. Economic integration with abolition of national customs tariffs causes dynamics of each state to be influenced by the value added of economy of its integrating counterpart. In other words, sectors' values added of a region previously separated from another neighboring one by tariffs, after their abolition will penetrate integrated counterpart and starts influencing the dynamics of the GDP of integrated state. Mathematically that means that we may use an interesting trick: right hand side of equation for the GDP dynamics of each state is now added by the value added components of the other integrating states.

As an example, let us consider the case of interaction of two sectors, an each in two integrated states. Equations of the GDP for countries 1 and 2 after economic integration become the following:

$$\ddot{Y}_1 + (2+s_1)\dot{Y}_1 + (1+s_1)Y_1 - \mu_1 \nabla^2 Y_1 = P_1Q_1 + P_2Q_2 + \dot{P}_1 Q_1 + P_1\dot{Q}_1 + \dot{P}_2 Q_2 + P_2\dot{Q}_2 = f_1, \quad (20)^6 \tag{20}$$

$$\ddot{Y}_2 + (2+s_2)\dot{Y}_2 + (1+s_2)Y_2 - \mu_2 \nabla^2 Y_2 = P_1Q_1 + P_2Q_2 + \dot{P}_1 Q_1 + P_1\dot{Q}_1 + \dot{P}_2 Q_2 + P_2\dot{Q}_2 = f_2 \quad (21) \tag{21}$$

Here $Y_i$ is the GDP of the region *i*, $f_k$ is a function of the values added of two united regions.

In order to understand qualitative behavior of $f_k$ we may use schematic analysis of the dynamics of price and output as components of the function $f_k$. On Figure 3 one may see cyclical co-movements of price *P* and output *Q*, with their oscillations identical to Volterra's "predator-prey" model, where the price is a "prey" and the output is "predator" (Dalimov, 2005):

$$\dot{P} > 0 \rightarrow \dot{Q} > 0 \rightarrow \dot{P} < 0 \rightarrow \dot{Q} < 0 \rightarrow \dot{P} > 0 \rightarrow \dot{Q} > 0 \ldots \tag{22}$$

One may see there that the value added function behaves as an oscillating one too. Let us conduct harmonic analysis of the forces $f_k$ using representation of price and output in the form of periodic functions:

$$P_1 = P_{10} \sin p_1 t, \tag{23}$$

$$P_2 = P_{20} \sin p_2 t, \tag{24}$$

and

$$Q_1 = Q_{10} \sin (q_1 t + \varphi_1), \tag{25}$$

$$Q_2 = Q_{20} \sin (q_2 t + \varphi_2), \tag{26}$$

Where $P_{i0}; Q_{i0}; p_k; q_k$ are corresponding amplitudes and frequencies of oscillations of prices and outputs; $\varphi_1$ and $\varphi_2$ are respective shifts of phases of oscillations. Then, using trigonometric identities, we obtain the following:

$$f = \frac{\partial}{\partial t}(P_{10} Q_{10} \sin p_1 t \sin(q_1 t + \varphi_1) + P_{20} Q_{20} \sin p_2 t \sin(q_2 t + \varphi_2)) + P_1Q_1 + P_2Q_2 =$$

$$= \frac{\partial}{\partial t}(\frac{P_{10}Q_{10}}{2}(\cos((p_1 - q_1)t - \varphi_1) - \cos((p_1 + q_1)t + \varphi_1) +$$

$$+ \frac{P_{20}Q_{20}}{2}(\cos((p_2 - q_2)t - \varphi_2) - \cos((p_2 + q_2)t + \varphi_2))) +$$

$$+ P_{10}Q_{10} \sin p_1 t \sin(q_1 t + \varphi_1) + P_{20}Q_{20} \sin p_2 t \sin(q_2 t + \varphi_2)$$

Frequencies $p_i$ and $q_i$ are approximately the same because of similar oscillations of price and output which differ by a phase only:

$$q_i - p_i \cong \varphi_i \tag{27}$$

In practice, phases $\varphi_i$ are quite small (see Figs. 3 and A1). Let $\varphi_i$ be close to nil. Then:

$$\omega_1 = p_1+q_1 = 2\ q_1 \tag{28}$$

$$\omega_2 = p_2+q_2 = 2\ q_2. \tag{29}$$

Hence, expression for the function of the value added becomes the following:

$$f = q_1P_{10}Q_{10}\sin 2q_1t + q_2P_{20}Q_{20}\sin 2q_2t + P_{10}Q_{10}\sin^2 q_1t + P_{20}Q_{20}\sin^2 q_2t =$$
$$= P_{10}Q_{10}(q_1\sin 2q_1t + 0.5(1-\cos 2q_1t)) + P_{20}Q_{20}(q_2\sin 2q_2t + 0.5(1-\cos 2q_1t)) =$$
$$= 0.5(P_{10}Q_{10}+P_{20}Q_{20})+0.5P_{10}Q_{10}(\omega_1\sin\omega_1t - \cos\omega_1t)+0.5P_{20}Q_{20}(\omega_2\sin\omega_2t - \cos\omega_2t) =$$
$$= \frac{1}{2}\sum_{i=1}^{2}P_{i0}Q_{i0}(1+\omega_i\sin\omega_it - \cos\omega_it),$$

where $\omega_i = 2q_i$. Thus, initial equations (20)-(21) are transformed to the following form:

$$\ddot{Y}_i + (2+s_i)\dot{Y}_i + (1+s_i)Y_i - \mu_i\nabla^2 Y_i = \frac{1}{2}\sum_{i=1}^{2}P_{i0}Q_{i0}(1+\omega_i\sin\omega_it - \cos\omega_it) \tag{30}$$

Variables may be separated using Fourier substitution $Y = Y_t(t)\cdot Y_s(x,y)$ in (30) and dividing both left and right sides to $Y_t(t)\cdot Y_s(x,y)$. In the same manner we assume that $f = f_t(t)\cdot f_s(x,y)$, where $f_t(t)$ and $f_s(x,y)$ are temporal and spatial components of the value added force. Finally, for the sake of simplicity we assume that $f_s(x,y) = aY_s(x,y)$, with $a = const$ thus excluding spatial dynamics from consideration since the main purpose of this article is to look for temporal dynamics of the GDP. Altogether we obtain:

$$\frac{\ddot{Y}_t}{Y_t} + (2+s)\frac{\dot{Y}_t}{Y_t} + (1+s) - \mu\frac{\nabla^2 Y_s}{Y_s} = \frac{f_t f_s}{Y_t Y_s}; \tag{31}$$

or

$$\frac{\ddot{Y}_t}{Y_t} + (2+s)\frac{\dot{Y}_t}{Y_t} + (1+s) - \frac{f_t}{aY_t} = \mu\frac{\nabla^2 Y_s}{Y_s} = \delta. \tag{32}$$

Since both parts of (30) are functions of independent variables, with the left one on time and the right one - on spatial coordinates, then equation (30) will be valid only in case when both sides are equal to the constant. This gives us two equations separately responsible for temporal and spatial dynamics of the GDP, $a$=1:

$$\begin{cases} \ddot{Y}_t + (2+s_i)\dot{Y}_t + (1+s_i+\delta_i)Y_t = \sum_{j=1}^{n}(P_jQ_j + \frac{\partial}{\partial t}P_jQ_j), \\ \nabla^2 Y_s + \frac{\delta}{\mu}Y_s = 0; \end{cases} \tag{33}$$

Where the former one is identical to the equation of enforced oscillations of the pendulum with friction. Having in mind these manipulations we return to analysis of (30): once again, due to specific property of enforced oscillations a variable $Y$ changes its amplitude and frequency to the ones of "external force". Regional GDP in (30) acts under influence of the value added functions of sectors of the region. In other words, to have an economic growth one needs to have oscillations of the GDP, which by nature of (30) takes place only in case of perpetual growth of terms in the right part of the equation.

Further analysis of (30) shows that each of the components in its right hand side makes its own contribution to the solution of a given equation. They are respectively the following:

a) For $0.5(P_{10}Q_{10}+P_{20}Q_{20})$:

$$Y_{ak}(t) = \frac{1}{2}\sum_{i=1}^{2}P_{i0}Q_{i0}(\frac{1}{1+s_k} + e^{-jt}\cos((1+s_k)t+\varphi_0))) \tag{34}$$

b) For $\sin\omega_i t$:

$$Y_{bk}(t) = \frac{1}{2}\sum_{i=1}^{2}\frac{P_{i0}Q_{i0}\omega_i}{\sqrt{(\omega_i^2-(1+s_k+\delta))^2+2(2+s_k)\omega_i^2}}\sin(\omega_it+\varphi_{ik}) \tag{35}$$

Where phase of oscillations is according to the solution of enforced oscillations:

$$tg\,\varphi_{ij} = \frac{\omega_i(2+s_j)}{\omega_i^2-(1+s_j+\delta_j)} \tag{36}$$

c) For $\cos\omega_i t$:

$$Y_{ck}(t) = -\frac{1}{2}\sum_{i=1}^{2}\frac{P_{i0}Q_{i0}}{\sqrt{(\omega_i^2-(1+s_k+\delta))^2+2(2+s_k)\omega_i^2}}\cos(\omega_it+\varphi_{ik}). \tag{37}$$

**Table 1.** Specific branches of the EU**.**

| № | Countries, *i* | Domestic savings rate, in % | Branches, *j* |
|---|---|---|---|
| 1 | **EU (15)** | 21,8 | Agriculture, hunting and forestry |
| 2 | **Hungary** | 21,2 | Fishing |
| 3 | **Cyprus** | 21,2 | Mining and quarrying |
| 4 | **Latvia** | 21,2 | Manufacturing |
| 5 | **Lithuania** | 21,2 | Electricity, gas and water supply |
| 6 | **Malta** | 21,2 | Construction |
| 7 | **Poland** | 21,2 | Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods |
| 8 | **Slovakia** | 21,2 | Hotels and restaurants |
| 9 | **Slovenia** | 21,2 | Transport, storage and communication |
| 10 | **Czech** | 21,2 | Financial services |
| 11 | **Estonia** | 21,2 | Real estate, renting and business activities |
| 12 | | | Education |
| 13 | | | Health and social work |

Second term in parenthesis of a) quickly nullifies. Therefore, for temporal component of the GDP, i.e. its temporal dynamics for the region *k,* we have the following final expression:

$$Y_k(t) = \frac{1}{2}\sum_{i=1}^{2} P_{i0}Q_{i0}\left(\frac{1}{1+s_k} + \frac{\omega_i \sin(\omega_i t + \varphi_{ik}) - \cos(\omega_i t + \varphi_{ik})}{\sqrt{(\omega_i^2 - (1+s_k+\delta))^2 + 2(2+s_k)\omega_i^2}}\right) \quad (38)$$

Where

$\omega_i = 2q_i;\ tg\varphi_j = \dfrac{2q_j(2+s_j)}{4q_j^2 - 1 - s_j - \delta_j}$ and $\delta_j$ is constant.

It is probably important to note that amplitude of oscillations for each sector has increased due to input of all sectors of integrated regions to "external force" for that particular sector. In other words, we just proved that international economic integration is beneficial to its participants. Expression (38) allows one to compute the magnitude of the GDP for integrated regions: one has to measure frequency of oscillations of the sectors' value added together with their maximal absolute values representing amplitudes of corresponding oscillations as well as to know the magnitude of domestic savings rate of the region and exact value of the magnitude $\delta$ .

## Modeling economic integration for 2004 EU expansion

We start it using expression for the regional GDP and letting $t = 0$, that is, we are going to compute GDP right at the moment of economic integration:

$$Y_k(t) = \frac{1}{2\sum_{i=1}^{13} v_{ij}} \sum_{i=1;j=1}^{i=13;j=2} P_{ij0}Q_{ij0}\left(\frac{1}{1+s_k} + \frac{\omega_{ij}\sin\varphi_{ijk} - \cos\varphi_{ijk}}{\sqrt{(\omega_{ij}^2 - (1+s_k+\delta))^2 + 2(2+s_k)\omega_{ij}^2}}\right) \quad (39)$$

Where the summation by *j* represents the contribution of regions, $v_{ij} = \dfrac{P_{ij}Q_{ij}}{Y_k(t_0)}$ ; and $t_0$ is a year previous to computed one.

In the given case it is the year 2004 as the moment of integration, after which in the year 2005 significant growth of the GDP's value was observed. Let us note that a similar increase in the rate of economic growth was previously mentioned in a number of articles (Bretschger and Steger, 2004; Badinger, 2001). The summation by *i* sectors was done according to Table 1. Schematic behavior of countries GDPs prior to and after economic integration is shown on Figures 4-5, where a horizontal plane stands for countries geographic, while vertical lines serve as tariffs of states.

Remarkably, one may see that there is straight forward analogy of values added waves' spatial allocation to the dynamics of waves in a water pool: the higher are boundaries, the less "waves" of values added may pass through it – and only specific sectors with magnitudes of the values added high enough may pass through the tariffs serving here as economic boundaries.

In case of one region the values added of its sectors seem to be circling in the economy: their picture is similar to waves in a water pool where we may have, e.g., two different waves generated by throwing simultaneously two stones on different places in the pool. Waves will start to co-interact, by-passing each other and

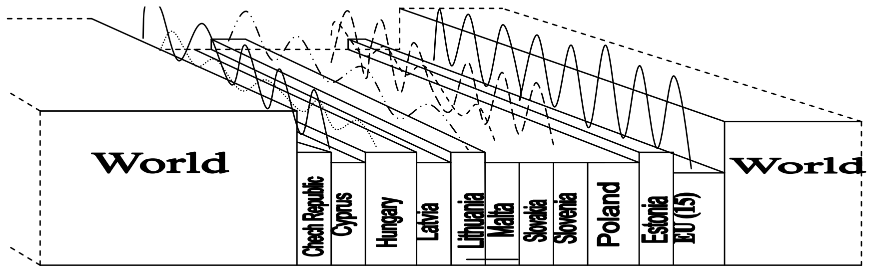



**Figure 4.** Pools of GDP's Oscillations prior to EU enlargement.

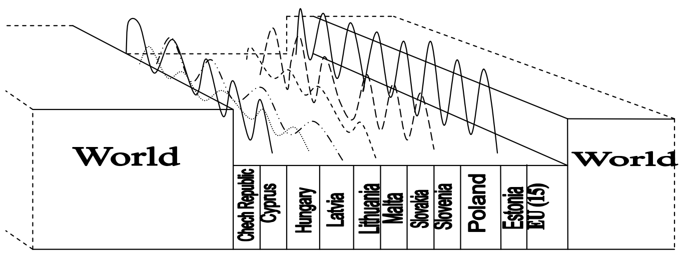



**Figure 5.** Common Pool of GDPs Oscillations after EU enlargement.

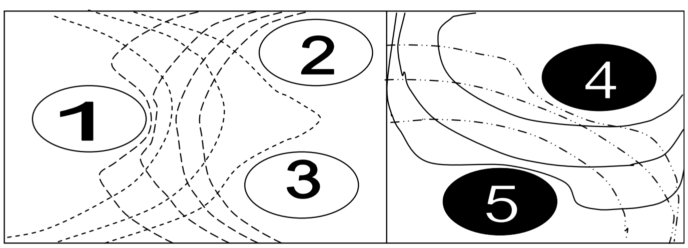



**Figure 6.** Intra-regional interaction of values added prior to integration.

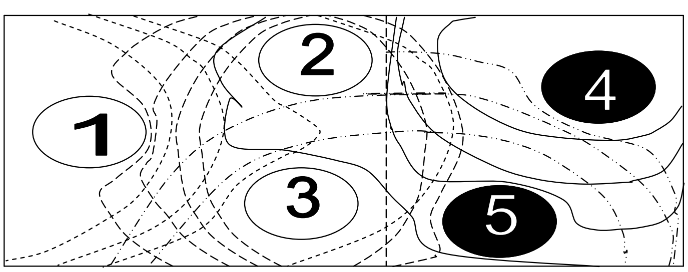



**Figure 7.** Inter-regional interaction of values added after integration.

simultaneously getting reflected from the boundaries of the pool; then there comes second by-passing of the reflected waves etc.

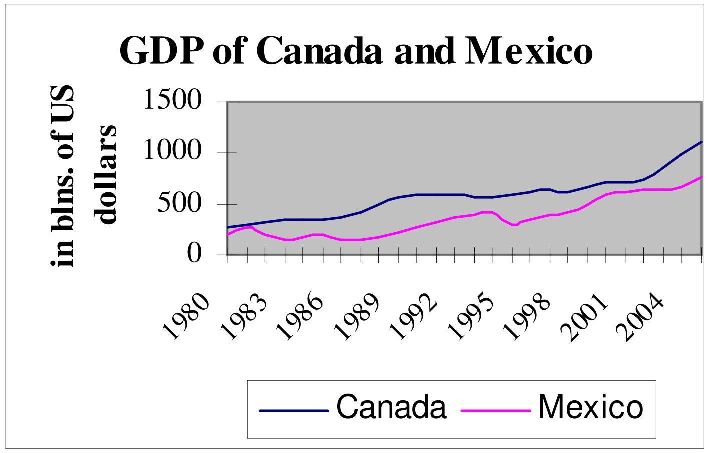
Now imagine several water pumps working in the pool, with waves of each of them cyclically spreading out (Figures 6 - 7). These water pumps in our case represent economic units: entities, sectors, manufacturers and traders, etc. Their number is as much as a number of economic units in the region, with all of them working out respective value added.

Economic integration of the regions previously separated by tariff and non-tariff restrictions in the sense we just mentioned looks like interaction of the values added waves of the different sets of economic units belonging to previously separate regions. Moreover, inside the region the sectors connected by an on-going process of creation and absorption of the values added are in a constant exchange of these "waves" of values added: in case of small absolute magnitudes they are reflected back inside the region, and when their amplitudes are high enough, they pass through the tariff barriers, thus representing successful exporting industries. These phenomena are well known in oscillation theory. Among its properties there is a fact that any change in the system of linked Oscillators influence the whole system, and not just its neighbors. One may note that any change in the economy: ups and downs - influences demand and supply of a whole economy, and not just one sector, which fits within the specified property of the model.

International economic integration comes at the moment of "annihilation" of trade barriers for values added - for all the unifying states simultaneously.

All manufacturers will be in a position of "sudden" increase of the influence by manufacturers in the economies, previously separate, e.g., from this particular state. This is due to the second summation by variable *j*: final magnitude of the GDP in (39) will always increase. To see the process of that increase in reality we shall return to Figure 2 where we may look at bigger ball as a bigger country's GDP, while small one stands for little country's GDP. For instance, the dynamics of the US economy when it entered NAFTA did not change much, since it was simply too big to notice the GDP's growth due to the creation of the free trade area, which was actually present, but comparatively small (GDP of Canada and Mexico made only 10% of the one for USA, Figure 8).

The same case is with 2004 EU expansion when 10 new states made only 5% addition to the size of the GDP for the EU-15. Economies of 10 new EU member states in 2005 grew for 21.5% while economies of the EU-15



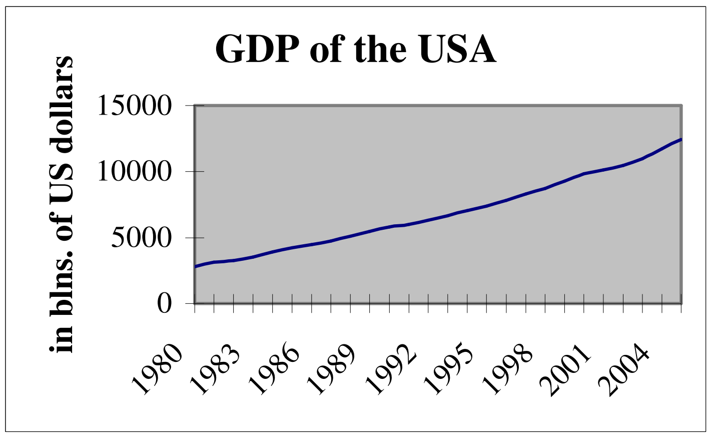




**Figure 8.** GDPs of NAFTA member states.

**Source:** World economic outlook, September, 2005. IMF database.

**Table 2.** Computed and real GDP of the EU countries prior / after economic integration in 2004, in mlns. of euro.

| | EU-15 | 10 new member states | EU-25 |
|---|---|---|---|
| GDP in 2004 | 9, 923, 079 | 485, 973 | 10, 409, 052 |
| GDP computed after integration in 2005 | 10, 338, 593 | 504, 491 | 10, 843, 084 |
| Real GDP in 2005 | 10, 137, 984 | 539, 448 | 10, 677, 432 |

**Source:** IMF database.

(old member states of the EU) grew only for 4.2%.

But when it comes to economic unification of comparatively equal countries we may expect notable increase in their GDPs (Table 2). These results obtained here analytically are not new in the theory of economic integration, which mentions higher growth of welfare for smaller states en-tering economic integration by their bigger counterparts while the latter ones get their benefits in the longer run, that is, without significant growth at the moment of econo-mic integration.

Methodology of calculations shall proceed through building the curves of the value added functions as multiplication of price by output *PQ* for each sector of the EU-15 and 10 new member states, which will allow measureing necessary frequencies and amplitudes of their historically last oscillations.

### Results of the modeling

Magnitude $\delta$ serves as a value showing an increase

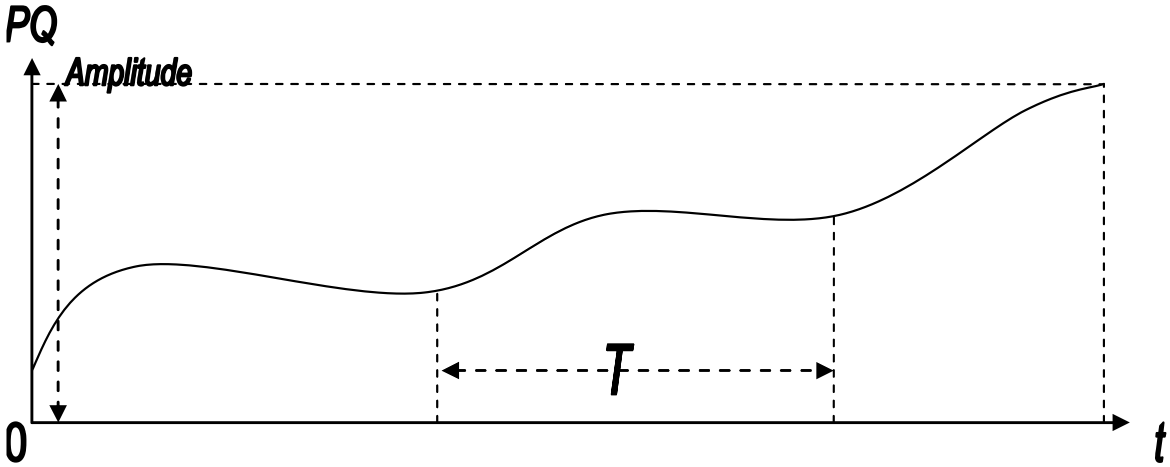



**Figure 9.** Dynamics of the value added and period of its oscillations.

in the value of the GDP. In equation of the GDP's temporal dynamics (Dalimov, 2006), the natural frequency of its oscillations is equal to $\omega_0^2 = 1 + s + \delta$. The higher is the natural frequency $\omega_0$, the more is the growth of the GDP.

Value of $\delta$ was found to be equal to 0.7525 from an inverted task of economic integration in case of Eurasian Economic Community (EvrAzEC) which was established in 2000 by Belarus, Kazakhstan, Kyrgyzstan, Russia and Tajikistan. Period of observations was 1992-2005: we had calculated $\delta$, knowing all the other parameters such as domestic savings rate, the values added as well as countries GDPs prior to and after economic integration.

During computation we had to take into account that the savings rates for the 10 new member states of the EU were equal, and as a consequence these states were considered as a whole one region. The computations show (Tables 2) that the aggregate GDP of the EU in 2005, that is, after integration, becomes 10,843,084 million euros while the real numbers for this GDP are 10,677,432 million euros: the difference between these values is about 1.6%.

Respective magnitudes for the GDP of the 10 new (2004) EU member states are 504,491 and 539,448 million euros, with a difference of 6.5% based on the level of the real GDP. The reason why the GDP of the EU-15 was raised only to 4.2%, while the GDP of the 10 new members to 21.5% was the magnitude of the GDP of the 10 new EU member states, more than 20 times less the size of the GDP of the EU-15.

## Conclusion

The article proposes new methodology in modeling international economic integration based on oscillation theory approach using fundamental purpose of economic integration - an increase of welfare in integrated states. At first, differential equation was deducted responsible for the dynamics of the GDP in the region. It was found to be similar to the case of enforced oscillations of attenuated pendulum. The "force" which influences the dynamics of the GDP is the value added function, analysis of which showed its cyclic nature.

Method of computing the GDP after economic integration proposed in the article shows its simplicity and ability to calculate benefits of economic integration in case of the creation an economic unions: for calculation one needs to know the rates of domestic savings, amplitudes of the willing to be united, as well as to find frequencies of value added functions for the countries / regions oscillations of the value added of sectors.

Oscillation theory approach gives clear picture on the dynamics of integrated regions, proposing a new insight to the theory and practice of international economic integration. Similarity of the dynamics of the value added of sectors within a region and during international economic integration to the wave dynamics in a pool raises a new agenda for further research in this area. The least scientists may do is to look for similar patterns between phenomena of non-linear wave dynamics e.g. solitons and the dynamics of the value added within a region. If successful, this promises to be a tool in qualitative forecasting of economic phenomena in market circumstances – really complex and currently unpredictable world.

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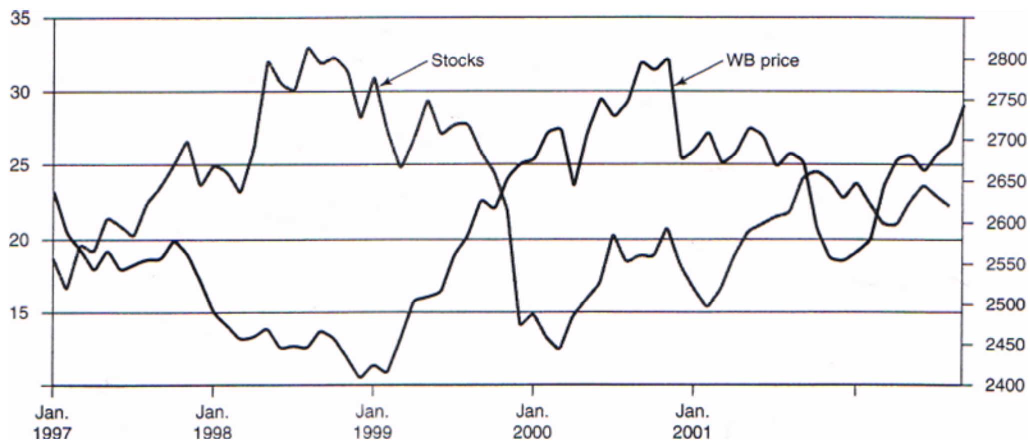
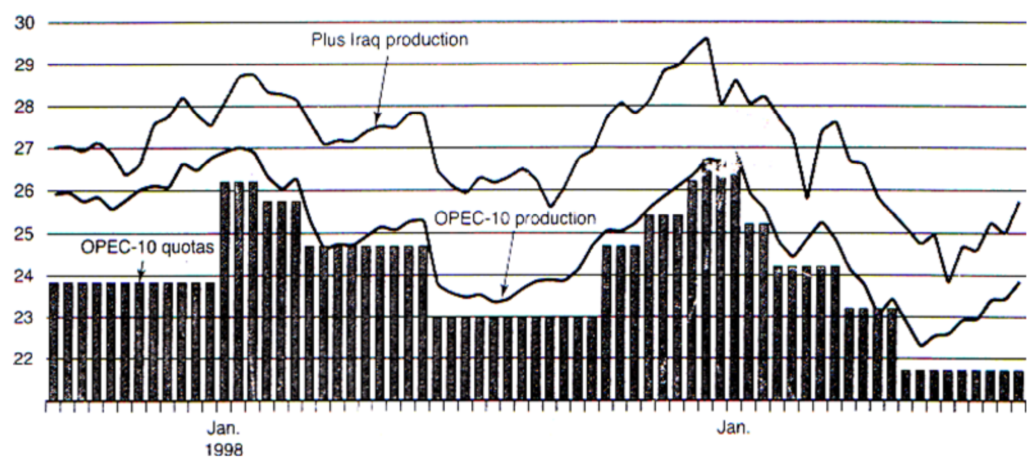
## Appendix

**Oil price and OECD stocks**
**US dollars per day**




Source: World Bank and International Energy Agency.

**OPEC-10 oil production**
**mln. barrels per day**




Source*:* International Energy Agency and Organization of Petroleum Exporting Countries

**Figure A1**. Correlated movements of price and output of oil in the world.