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On Pearson families of distributions and its applications

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In this study we are going to discuss an extended form of Pearson, including the reversed generalized Pearson curves distribution as its subfamily, and refer to it as the extended generalized same distribution. Because of many difficulties described in the literature in modeling the parameters, we propose here a new extended model. The model associated to this heuristic is implemented to validate the result of the generalized Pearson family routine in the specific cases. This study is presents same applications of Pearson family's of distributions, and give the new extended, which extends the classical Pearson family. Various properties of this new family are investigated and then exploited to derive several related results, especially characterizations, in probability. As a motivation, the statistical applications of the results based on health related data are included. It is hoped that the findings of this work will be useful for the practitioners in various fields of theoretical and applied sciences.

Key words: Pearson and Burr family distribution extended, special, families, goodness of fit.

INTRODUCTION

In his original paper, Pearson (1895) identified four types of distributions (numbered I through IV) in addition to the normal distribution (which was originally known as Type V). Rhind (1909) devised a simple way of visualizing the parameter space of the Pearson system, which was subsequently adopted by Pearson (1916) (Rhind, 1909). The Pearson types are characterized by two quantities commonly referred to as β_1 and β_2. A Pearson density $f(x)$ is defined to be any valid solution to the differential equation (Pearson, 1895):

$$\frac{df(x)}{dx}+\frac{a+(x-\lambda)}{d(x-\lambda)^2+c(x-\lambda)+b}=0, \tag{1}$$

$$with \quad \begin{cases} b=\dfrac{4\beta_2-3\beta_1}{10\beta_2-12\beta_1-18}\mu_2, \\ a=c=\sqrt{\mu_2}\sqrt{\beta_1}\dfrac{\beta_2+3}{10\beta_2-12\beta_1-18}, \\ d=\dfrac{2\beta_2-3\beta_1-6}{10\beta_2-12\beta_1-18} \end{cases}$$

The Pearson family of distributions was designed by Pearson between 1890 and 1895. It represents a system whereby for every member the probability density function $f(x)$ is composed of twelve families of distributions, all of which are solutions to the differential equation (Pearson, 1895).

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$$\frac{df(x)}{dx} = \frac{(x-a)f(x)}{dx^2 + cx + b}, \qquad (2)$$

The solutions differ in the values of the parameters a, b, c and d. The Pearson system includes the normal, Gamma, and Beta distributions among the families. The twelve families cover the entire skewness and kurtosis plane. The gamma distributions are also referred to as Pearson Type III distributions, and they include the chi-square, exponential, and Erlang distributions, and the beta distributions are also referred to as Pearson Type I or II distributions (Bagnoli and Bergstrom, 2005).

A generalization of the Pearson differential Equation (2) has appeared in the literature, from which a vast majority of continuous probability density functions can be generated, known as the generalized Pearson system of continuous probability distributions (Mohammad et al., 2010).

PEARSON FAMILIES OF DISTRIBUTIONS

This family of distributions based on the solutions of Pearson differential Equation (1). The family was proposed by Karl Pearson in 1894 as a response to his recognition that not all populations had distributions that resembled the normal distribution. He proposed twelve types of distribution which are variants of three basic distributions. The Pearson family of distributions is made up of seven distributions: Type I to VII. It covers any specified average, standard deviation, skewness and kurtosis. Together they form a 4-parameter family of distributions that covers the entire skewness-kurtosis region other than the impossible region. The seven types are described below:

(i) Type I: Beta Distribution
(ii) Type II: Special case of beta distribution that is symmetrical.
(iii) Type III: Gamma Distribution.
(iv) Type IV: Region above Type V.
(v) Type V: 3 parameter distribution represented by curve
(vi) Type VI: Region between Gamma and Type V.
(vii) Type VII: Special case of Type IV that is symmetrical

The special cases can be ignored (II, VII) and (I, III) types are alias for distribution already covered. That leaves Type IV, V and VI as new distributions. The diagram of different distributions of Pearson curves family. Here β_1=skewness 2; β_2=kurtosis+3. The moments of GS-distributions of different lines represent combinations of all parameters and their relation to third and fourth moments through: $\beta_1 = \mu_3^2/\mu_2^3$ (Pearson and Hartley, 1972). All the distributions below the line representing the Type V distributions belong to the Type IV family, and therefore, Pearson Type IV distributions.

cover a wide region in the skewness-kurtosis plane (Figures 1 and 2). The well-known families of continuous probability distributions such as the normal and student t distributions (known as Pearson Type VII), beta distributions (known as Pearson Type I), and gamma distributions (known as Pearson Type III) (Mohammad et al., 2010).

Elderton (1907) gave a systematic description of the types of Pearson curves. In simplified form, the classification by types is as follows:

Type I: The distribution function is:

$$f(x) = k\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_2}\right)^{m_2}, -a_1 \prec x \prec a_2; m_1, m_2. \succ 0 \qquad (3)$$

with as a particular case the beta distribution of the first kind.

Type II: The distribution function is:

$$f(x) = k\left(1-\frac{x^2}{a^2}\right)^{m}, -a \prec x \prec a \; ; m \succ -1 \qquad (4)$$

(a version of a Type-I Pearson curve); a particular case is the uniform distribution).

Type III: The distribution function is:

$$f(x) = k\left(1+\frac{x}{a}\right)^{\mu a} \exp\{-\mu x\}, -a \prec x \prec +\infty; \mu, a \succ -1, \qquad (5)$$

Particular cases are the gamma distribution and the chi squared distribution.

Type IV: The distribution function is:

$$f(x) = k\left(1+\frac{x^2}{a^2}\right)^{-m} \exp\{-\mu\, arctg(x/a)\}, -\infty \prec x \prec \infty; a, \mu, m \succ 0,. \qquad (6)$$

Type V: The distribution function is:

$$f(x) = kx^{-p} \exp\{-(\alpha/x)\}, 0 \prec x \prec \infty; \alpha \succ 0, p \succ 1, \qquad (7)$$

(which can be reduced by transformation to Type III).

Type VI: The distribution function is:

$$f(x) = kx^{-p}(x-a)^q, a \prec x \prec \infty; p \prec 1, q \succ -1, p \succ q-1, \qquad (8)$$

Particular cases are the beta-distribution of the second kind and the Fisher (F-distribution).

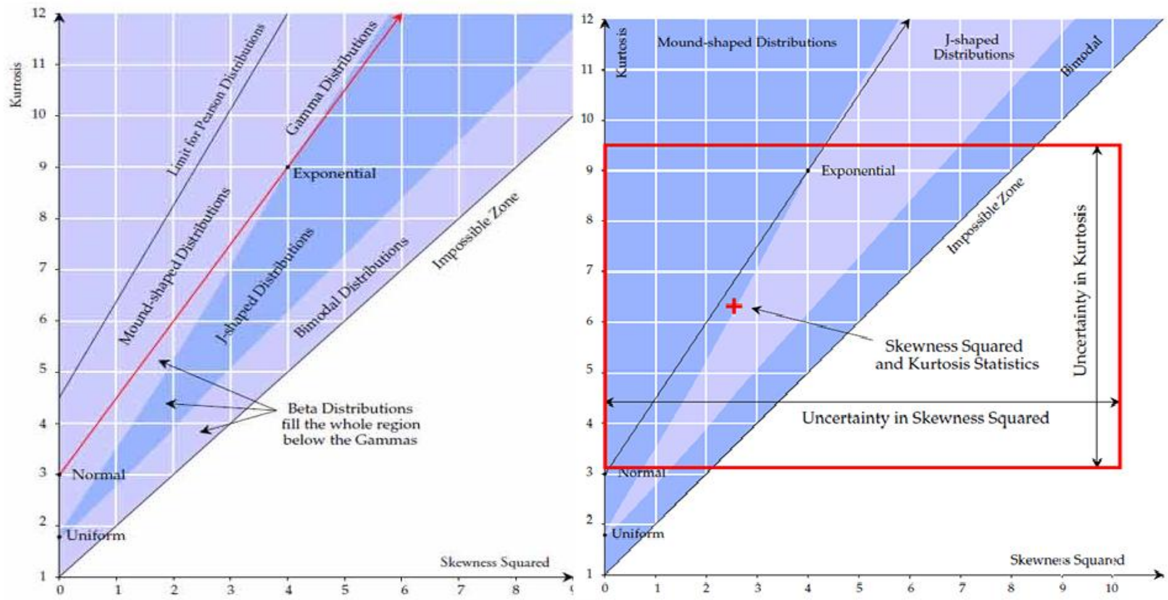

Figure 1. Distributions cover a wide region in the skewness–kurtosis plane.

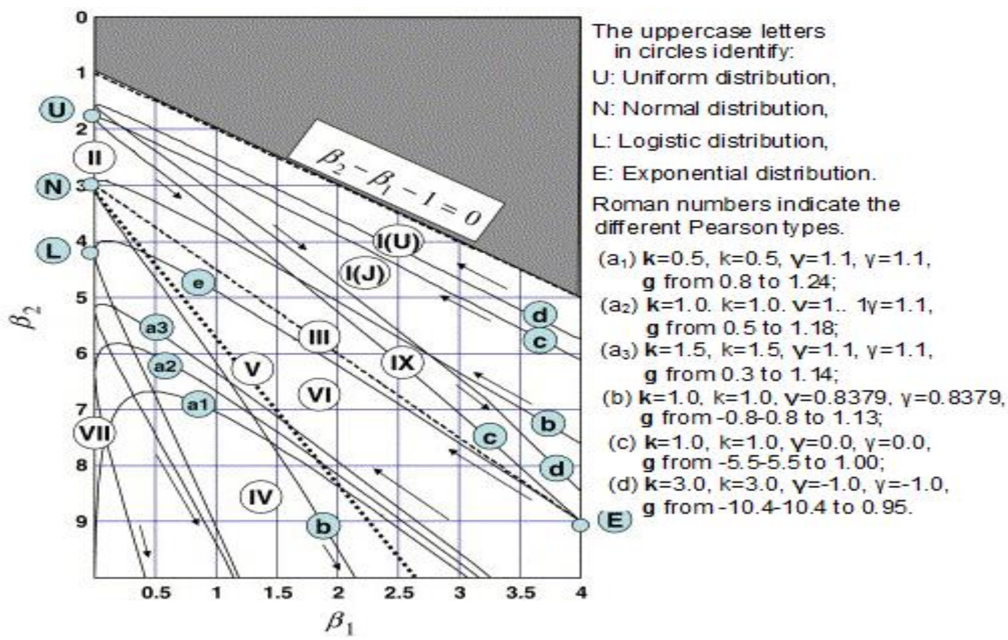

Figure 2. Pearson family distributions cover a wide region in β_1 and β_2 plane.

Type VII: The distribution function is:

$$f(x) = k\left(1+\frac{x^2}{a^2}\right)^{-m}, -\infty \prec x \prec \infty; m \succ 1/2, \quad (9)$$

A particular case is the Student distribution.

Type VIII: The distribution function is:

$$f(x) = k\left(1+\frac{x}{a}\right)^{-m}, -a \prec x \le 0; m \succ 1, \quad (10)$$

Type IX: The distribution function is:

$$f(x) = k\left(1+\frac{x}{a}\right)^m, -a \prec x \le 0; m \succ -1, \quad (11)$$

Type X: The distribution function is:

$$f(x) = k \exp\{-(x-m)/\sigma\}, m \le x \prec \infty; \sigma \succ 0, \quad (12)$$

That is, an exponential distribution.

Type XI: The distribution function is:

$$f(x) = kx^{-m}, b \le x \prec \infty; m \succ 1 \quad (13)$$

a particular case is the Pareto distribution.

Type XII: The distribution function is:

$$f(x) = \left(1+\frac{x}{a}\right)^m \left(1+\frac{x}{b}\right)^{-m}, -a \prec x \prec b; |m| \prec 1,. \quad (14)$$

(a version of Type I). The most important distributions for applications are the Types I, III, VI, and VII. This study derives a new family of distributions based on the generalized inverse Gaussian distribution. Some characteristics of new distribution are obtained (Mohammad et al., 2010).

EXTENDED GENERALIZED PEARSON FAMILIES OF DISTRIBUTIONS

The classical differential equation introduced by Karl Pearson during the late 19th century is a special case (Mohammad et al., 2010). For details on the Pearson system of continuous probability distributions, the interested readers are referred to Elderton and Johnson (1969) and Johnson et al. (1994), among others.

The new extended generalized Pearson family of distributions is characterized by general differential equation and is defined with implicit form. A random variable X that has a probability density function is said to have an extended generalized Pearson distribution of the following form:

$$\frac{df_X(x)}{dx} = g(x) f_X(x); \quad (15)$$

where $g(x)$ is integral real function, in particular case

$$g(x) = \frac{P_n(x)}{Q_m(x)}, \quad P_n(x) = \sum_{i=1}^{n} a_i x^i, \quad Q_m(x) = \sum_{i=1}^{m} b_i x^i$$

(Shakil et al., 2010b).

In terms of the model function statistical properties; estimation of some parameters of the distribution are studied. The Equation (15) has its most general form is:

$$g(x) f_X(x)dx - df_X(x) = 0,$$

Equation (15) is a separable differential equation is a first-order ordinary equation that is algebraically reducible to a standard differential form in which each of the non-zero terms contains exactly one variable solution to this kind of equation is usually quite straightforward. For example, the solution will be, of course we can obtain the general solution of above equation as follows:

$$\int \frac{df_X(x)}{f_X(x)} - \int g(x)dx = c \quad (16)$$

One way of discovering whether or not a given equation is separable is to collect coefficients on the two differentials and see if the result can be put in the form:

$$\ln\left(f_X(x)\right) = \int g(x)dx + c$$

Another way is to solve for a derivative and compare the result with

$$\left(f_X(x)\right)' = g(x) f_X(x);$$

A general solution of the form can be found by first dividing by the product function to separate the variables and then integrating can be solved by first multiplying and subsequently integrating:

$$f_X(x) = \exp\left\{\int g(x)dx + c\right\} = \lambda \exp\left\{\int g(x)dx\right\} \quad (17)$$

The process of solving a separable equation will often involve division by one or more expressions. In such cases the results are valid where the divisors are not equal to zero but may or may not be meaningful for values of the variables for which the division is undefined. Such values require special consideration and may lead to singular solutions. It implies from the definition of X that $f_X(x) \ge 0, -\infty \le x < \infty$, and we have $\int_{-\infty}^{\infty} f_X(x)dx = 1$.

An extended generalization of the Pearson differential equation has appeared in the different literature, known as the generalized Pearson system of continuous probability distributions. Equation (17) derives a new family of distributions based on the generalized Pearson differential equation.

It is observed that the new distribution is skewed to the right and bears most of the properties of skewed distributions. Equation (17) develops some new classes of continuous probability distributions based on the generalized Pearson differential Equation (15). Some characteristics of the new distributions are defined. Some different families of distributions based on generalized Pearson differential Equations (1 to 12). One of these systems is the Pearson system.

PEARSON, BURR AND JOHNSON SYSTEMS

Probability integrals and percentage points of univariate distributions from up to eight different families, having common first four moments are compared. The Burr distribution has a flexible shape and controllable scale and location which makes it appealing to fit to data. Burr (1942) chose to work with cdf $F(x)$ satisfying the Burr equation.

The distribution name comes from Johnson (1949) who proposed a system for categorizing distributions, in much the same spirit that Pearson did. Johnson's idea was to translate distributions to be a function of a unit normal distribution, one of the few distributions for which there were good tools available at the time to handle (Johnson, 1965).

It is frequently used to model insurance claim sizes, and is sometimes considered as an alternative to a normal distribution when data show slight positive skewness. Among interesting observations is the remarkable consistency in the standardized upper and lower points over considerable regions of the β_1 and β_2 plane; also the closeness of agreement between the log-normal and non-central t distributions and the Pearson Type VI and Type IV curves respectively (Shao et al., 2004).

The Johnson bounded distribution has a range defined by the min and max parameters. Combined with its flexibility in shape, this makes it a viable alternative to the pert, triangle and uniform allows the user to define the bounds and pretty much any two statistics for the distribution (mode, mean, standard deviation) and will return the corresponding distribution parameters.

We have discussed and introduced the continuous versions of the Pearson family, also found the log-concavity for this family in general cases, and then obtained the log-concavity property for each distribution that is a member of Pearson family. For the Burr family these cases have been calculated, even for each distribution that belongs to Burr family. Also, log-concavity results for distributions such as generalized gamma distributions, Feller-Pareto distributions, generalized inverse Gaussian distributions and generalized Log-normal distributions have been obtained. The plots for the cdf, pdf and hazard function, percentile points and tables for Pearson's measures of skewness and kurtosis for selected coefficients and parameters have been provided (Mohammad et al., 2010).

It has also been observed that a number of other distributions in Figure 3 including those of Chou and Huang (2004), among others. In what follows, some characteristics of our newly proposed distribution, including the percentile points, for some selected values of parameters, have been provided (Chou and Huang, 2004). They quite rightly pointed out that the extended Burr XII distribution is also known as the generalized Weibull distribution. The suggested reference of Mudholkar and Huston (1996) provides the reader with another perspective for considering our proposed extended Burr XII distribution.

The generalized Weibull distribution, originally named in Mudholkar and Huston (1996), is the same as what we called the extended Burr XII distribution, where the subscripts denote the named distribution-GW: generalized Weibull and EBXII: extended Burr XII. The Weibull distribution may be generalized in various ways and different generalizations may in fact be called the generalized Weibull distribution. Sometimes a generalized Weibull may be obtained by generalizing another distribution such as the Burr XII distribution, and the name reflects this perspective.

More importantly, our study is mainly to promote the use of the extended Burr XII distribution (which can also be called the generalized Weibull distribution) in the flood frequency analysis. We provided the evidence of its relationship with several popularly used distributions in flood frequency analysis, although there are other related distributions. Examples of the Pearson Types V, VI distributions are shown in Figure 4 (Shao et al., 2004). The Burr distribution is a right-skewed distribution bounded at the minimum value of a. b is a scale parameter while c and d control its shape Burr (0, 1, c, d) is a unit Burr distribution. Examples of the Burr distribution are shown in Figure 5 (Yuichi, 1999; Turner, 1962).

The Johnson bounded distribution has a range defined by the min and max parameters and will return the corresponding distribution parameters. Examples of the Johnson distribution are given in Figure 6.

FITTING PEARSON FAMILIES OF DISTRIBUTIONS

When Pearson developed his family of distributions, the inverse cumulative distribution function method can be used to generate random variables. Pearson used the method of moments, which is not really adequate in many cases, but many be used to provide starting values to maximum likelihood fitter.

Those interested in the method should read the text in Tukey lambda distribution. This is accomplished by computing the inverse of the C.D.F evaluated at a uniform random variable (Brawn and Upton, 2007). Integrating the P.D. or the exponential function $f_X(x) = \lambda \exp\left\{\int g(x)dx\right\}$, results in the C.D.F:

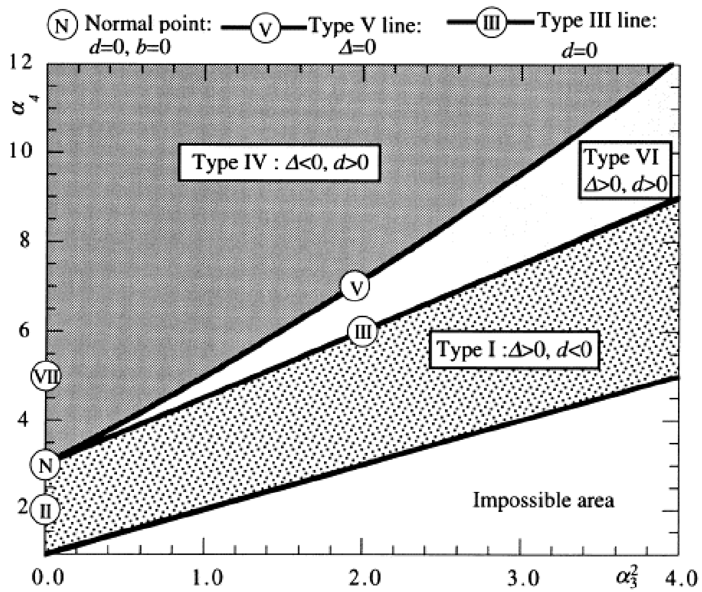

Figure 3. Distributions cover a wide region in the Pearson type plane.

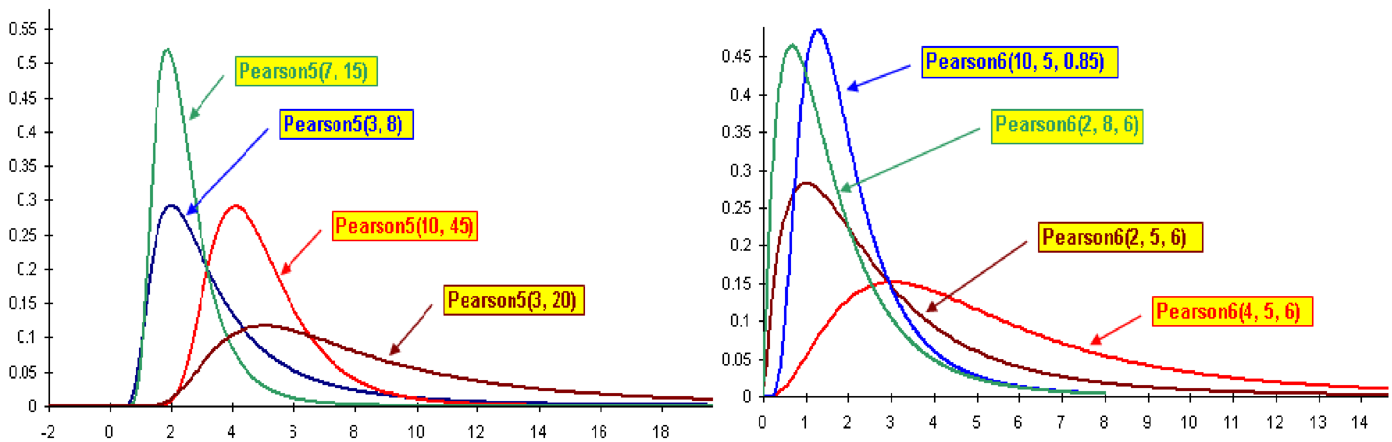

Figure 4. The Pearson Types V, VI distributions with same parameters.

$$F(x) = \int_0^x \lambda \exp\left\{\int g(t)dt\right\}dt$$

The inverse can be found performing by the following steps:

$$x = F(F^{-1}(x))$$

The random variables having density $f(x)$ may be generated by repeatedly calculating $F^{-1}(U)$ for values of U with a uniform density between 0 and 1 (Brawn and Upton, 2007). This function can be used for the untruncated case, however, we will need a modified version that accounts for λ in order to generate random variables for the truncated case $x = \tilde{F}(\tilde{F}^{-1}(x))$.

The results from this program can then be compared to the calculated values derived from the procedure of implementing the Log Pearson Type III distribution.

The Pearson Type III distribution is one of seven types of distributions devised by Karl Pearson (1895), a British statistician, Pearson distributions during the 1970's. The estimation of parameters by maximum likelihood estimation and method of fitting are discussed (Mohammad et al., 2010).

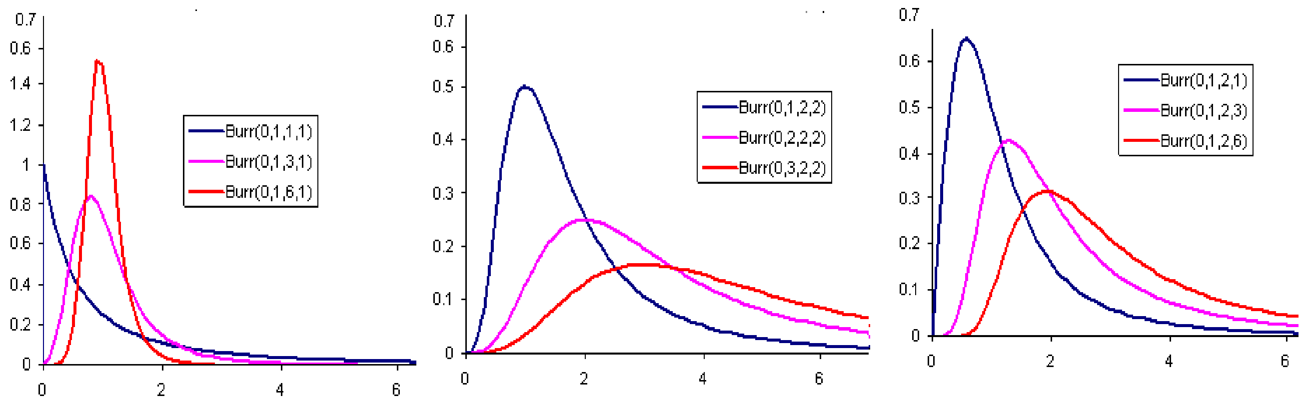

Figure 5. The Burr distribution Burr (0, 1, c, d) is a unit with same parameters.

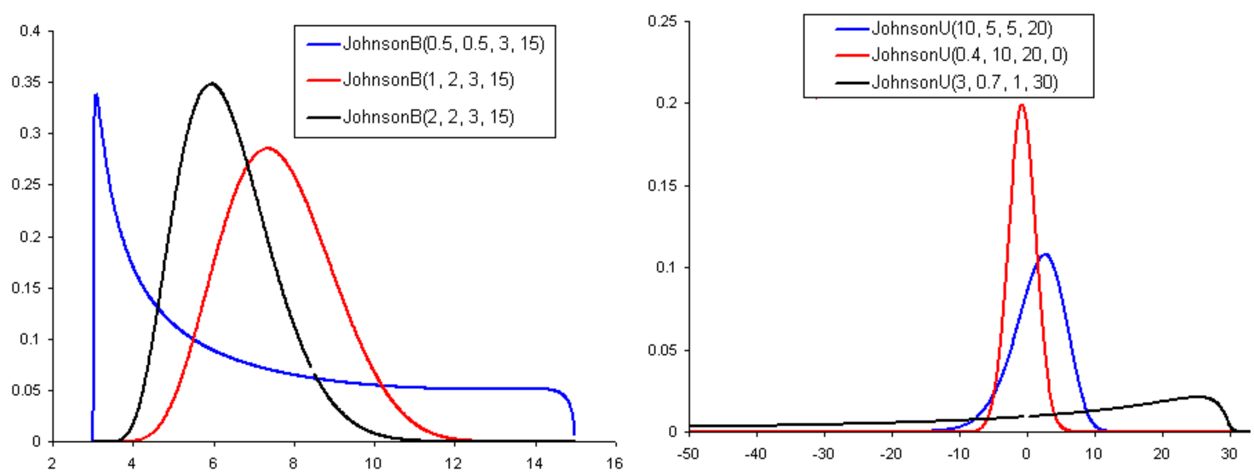

Figure 6. The Johnson distribution B (a_1, a_2, min, max, U) with same parameters.

APPLICATIONS

The application of Pearson system has advantages in capability to many researchers have studied the uses of the inverse Gaussian (IG) and the Generalised Inverse Gaussian (GIG) distributions in the fields of biomedicine, demography, environmental and ecological sciences, lifetime data, reliability theory, traffic data, etc (Dadpay et al., 2007).

The selection of a specific statistical distribution as a model for describing the population behavior of a given variable is seldom a simple problem. One strategy consists in testing different distributions (normal, lognormal, Weibull, etc.), and selecting the one providing the best fit to the observed data and being the most parsimonious (Yuichi, 1999).

This family of distributions is used to derive a closed-form expression for the Burr system. Many nice properties similar to those of multivariate normal distributions and allow analyzing scenarios where Gaussianity assumption no longer applies. Bagnoli and Bergstrom (2005) have obtained log-concavity for distributions such as normal, logistic, extreme-value, exponential, Laplace, Weibull, power function, uniform, gamma, beta, Pareto, log-normal, Student's t, Cauchy and F distributions (Johnson, 1965).

We have discussed and introduced the continuous versions of the Pearson family, also found the log-concavity for this family in general cases, and then obtained the log-concavity property for each distribution that is a member of Pearson family. For the Burr family, these cases have been calculated, even for each distribution that belongs to Burr family. Also, log-concavity results for distributions such as generalized gamma distributions, Feller-Pareto distributions, generalized inverse Gaussian distributions and generalized Log-normal distributions have been obtained. These models are used in financial markets, given their ability to be parameterised in a way that has intuitive meaning for market traders. A number of models are in current uses that capture the stochastic nature of the volatility of rates, stocks etc. and this family of distributions may prove to be one of the more important (Carptenter and Shenton, 1964). In the United States, the Log-Pearson III is the default distribution for flood frequency analysis (Lee, 2010).

A new extended generalized Pearson distribution which can be used in modelling survival data, reliability problems and fatigue life studies is introduced. Its failure rate function can be constant, decreasing, increasing, upside-down bathtub or bathtub-shaped depending on its parameters. It includes as special sub-models the exponential distribution, the generalized exponential distribution (Gupta and Kundu, 1999). Generalized exponential distributions and the extended exponential distribution (Gupta and Kundu, 1999).

A comprehensive account of the mathematical properties of the new family of distributions is provided. Close form fitting of the unknown parameters of the new model for complete sample as well as for censored sample is discussed. Estimation of the lambda parameter distribution is also considered (Brawn and Upton, 2007).
After classifying the different qualitative behaviors of the S-distribution in parameter space, we show how to obtain different families of distributions that accomplish specific constraints. One of the most interesting cases is the possibility of obtaining distributions that acomplish P(X ≤ Xc) = 0. Then, we demonstrate that the quantile solution facilitates the use of same distributions in Tukey Lambda distributions experiments through the generation of random samples (Ramberg et al., 1979). Use Inverse closed form we obtain an analytical solution for the quantile equation that highly simplifies the use of Pearson families of distributions. We show the utility of this solution in different applications.

Alternatively, one can make a choice based on theoretical arguments and simply fit the corresponding parameters to the observed data. In either case, different distributions can give similar results and provide almost equivalent models for a given data set. Model selection can be more complicated when the goal is to describe a trend in the distribution of a given variable. In those cases, changes in shape and skewness are difficult to represent by a single distributional form (Shakil et al., 2010a).

As an alternative to the use of complicated families of distributions as models for data, these distributions provide a highly flexible mathematical form in which the density is defined as a function of the cumulative. New extended Pearson families of distributions provide an infinity of new possibilities that do not correspond with known classical distributions (Seshadri and Wesolowski, (2001). Although the utility and performance of this general form has been clearly proved in different applications, its definition as a differential equation is a potential drawback for some problems (Hernandez-Bermejo and Sorribas, 2001). Finally, we show how to fit a new family of distributions to actual data, so that the resulting distribution can be used as a statistical model for them.

EXAMPLE OF PEARSON GOODNESS OF FIT TEST FOR A POISSON DISTRIBUTION

Table 1 presents count data on the number of Larrea divaricata plants found in each of 48 sampling quadrants, as reported in the Study "Some sampling characteristics of plants and arthropods of the Arizona desert," (Jay, 1962) (Turner, 1962). For cells 1, 2, 3, 4, and 5, the respective observed cell counts are 9, 9, 10, 14, and 6.

Let Y = number of plants in a quadrant. Assume that Y has a Poisson distribution. We will assume for the moment that the six counts in cell 5 were actually 4, 4, 5, 5, 6, and 6 Based on the observed data, the maximum Likelihood estimation for the Poisson parameter is the mean of the sample data (Lee, 2010),

$$\hat{\lambda} = \frac{(0)(9)+(1)(9)+(2)(10)+(3)(14)+(4)(2)+(5)(2)+(6)(2)}{9+9+10+14+2+2+2} = 2.10 \cdot$$

We want to test whether a Poisson distribution is a good fit to the data, using Pearson's chi-squared test and the estimated value of the Poisson parameter. We have n = 48, and the expected cell counts are $\mu_j = n\pi_j(\hat{\lambda})$, for j = 1, 2, 3, 4, 5. From the Poisson distribution, we have;

$$\pi_1(\hat{\lambda}) = P(Y=0) = \frac{e^{-2.10}(2.10)^0}{0!} = e^{-2.10},$$

$$\pi_2(\hat{\lambda}) = P(Y=1) = \frac{e^{-2.10}(2.10)^1}{1!} = 2.10e^{-2.10},$$

$$\pi_5(\hat{\lambda}) = P(Y=2) = \frac{e^{-2.10}(2.10)^2}{2!} = 2.205e^{-2.10},$$

$$\pi_4(\hat{\lambda}) = P(Y=3) = \frac{e^{-2.10}(2.10)^3}{3!} = 1.5435e^{-2.10}, \quad \text{and}$$

$$\pi_5(\hat{\lambda}) = P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - e^{-2.10}(1 + 2.10 + 2.205 + 1.5435) \cdot$$

Then the expected cell frequencies are

$$\mu_1 = n\pi_1(\hat{\lambda}) = 5.8779, \qquad \mu_2 = n\pi_2(\hat{\lambda}) = 12.3436,$$
$$\mu_3 = n\pi_3(\hat{\lambda}) = 12.9608, \qquad \mu_4 = n\pi_4(\hat{\lambda}) = 9.0726,$$
$$\mu_5 = n\pi_5(\hat{\lambda}) = 7.7451.$$

We want to test the null hypothesis that the cell probabilities are those found from a Poisson (2.10) distribution v. the alternative hypothesis that they are not;

Table 1. Data on the number of *Larrea divaricata* plants.

Observed counts					
Cell	1	2	3	4	5
Number of plants	0	1	2	3	≥ 4
Frequency	9	9	10	14	6

we will use α = 0.05. The test statistic is Pearson's chi-squared statistic:

$$X^2 = \sum_{j=1}^{5} \frac{\left(n_j - n\pi_j\left(\hat{\lambda}\right)\right)^2}{n\pi_j\left(\hat{\lambda}\right)},$$ which under the null hypothesis has a $\chi^2(3)$ distribution. The critical value for the test is $\chi^2_{0.05}(3) = 7.815$. The value of the test statistic calculated from the data is

$$X^2 = \sum_{j=1}^{5} \frac{\left(n_j - n\pi_j\left(\hat{\lambda}\right)\right)^2}{n\pi_j\left(\hat{\lambda}\right)} = \frac{(9-5.8779)^2}{5.8779} + \frac{(9-12.3436)^2}{12.3436} + \frac{(10-12.9608)^2}{12.9608} + \frac{(14-9.0726)^2}{9.0726} + \frac{(6-7.7451)^2}{7.7451} = 6.3097 \cdot$$

Since this value is less than the critical value of the test, we fail to reject the null hypothesis at the 0.05 level of significance. We do not have sufficient evidence to conclude that the data do not fit a Poisson (2.10) distribution.

CONCUDING REMARKS

The infinite divisibility property of the extended model proposed by this study is not completely discussed. It is observed in the new distribution that most of properties of skewed distributions are oriented to the right and have been provided in some cases. Some characteristics of the newly proposed distributions are obtained in this model. It is found that the general distribution of Pearson family show some cases of gamma, log-normal and inverse gamma distributions.

The existence of finite tails is of practical relevance for computations involving GS-distributions, for instance, integration below the left endpoint when some values of parameters lead to computational errors. Similar results apply for heavy left tails, which are associated with values of same parameters.

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