

Full Length Research Paper

Reliability and economic analysis of 2-out-of-3 redundant system with priority to repair

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Two reliability models have been developed to compare some reliability and economic measures for a 2-out-of-3 redundant system having two original units (called standard units) and one duplicate unit which are initially taken as spare in cold standby. The system is considered in up-state if any two of original and/or duplicate units are operative in parallel mode. The original unit is inspected at its failure by the server to see the feasibility of repair. If repair is not feasible, it is replaced by a new original unit. However, repair of the duplicate unit is done without inspection. In model II, priority to repair the original unit over the duplicate unit is given. The failure time of the unit(s) is exponentially distributed while inspection and repair time distributions are assumed as arbitrary with different probability density functions. The semi-Markov process and regenerative point technique have been used to derive the expressions for mean time to system failure and profit of the models. A particular case is considered to highlight the results graphically.

Key words: 2-out-of-3 system, inspection, priority to repair, reliability and economic measures.

INTRODUCTION

The stochastic models of redundant systems have widely been studied in the field of reliability as these are frequently used in modern technology. Various researchers including Srinivasan and Gopalan (1984), Singh (1989), Gupta and Chaudhary (1994), Tuteja and Malik (1994) have analysed systems of identical units with different repair policies under the common assumption that each unit can perform the required functions well. But in case of high cost of a unit, the duplicate unit might be kept as spare to use in emergency. Malik and Singh (2005) evaluated profit for reliability models of an electric transformer and generator. In many electrical and mechanical systems partial redundancy has also been used. In particular, there are systems of three units in which two units are sufficient to perform functions and one unit may be kept as spare in cold standby. The communication system with three transmitters can be cited as a good example of such systems where the messages load may be such that at least two transmitters must be operational at all times otherwise critical message will be lost. Chander

and Bhardwaj (2007) analysed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Further, sometimes it becomes necessary to give priority in repair to one unit over another unit. Chander (2005) probed reliability models of non-identical units with priority for operation and repair.

In view of practical applications of three unit systems, here two reliability models for a 2-out-of-3 redundant system are developed under different sets of assumptions. The system has two original units (called standard units) which work in parallel and one duplicate unit taken as spare in cold standby which can be used in emergency. The system is considered in up-state if any two original and/ or duplicate units are operative. The original unit is inspected at its failure by the server to see the feasibility of repair. If repair is not feasible, it is replaced by the new original unit. However, repair of the duplicate unit is done without inspection. In model II, priority to repair the original unit over the duplicate unit is given. The distribution of failure time of the units is taken as exponential while inspection and repair times are distributed arbitrarily. The random variables are mutually independent and uncorrelated. Switch over is instantaneous and repairs are assumed as perfect. The expressions for

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some reliability and economic measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period, expected number of visits by the server and profit function of the models have been derived using semi-Markov process and regenerative point technique. The results for a particular case have also been obtained to compare and depict the behaviour of MTSF and profit of the models graphically.

Notations

E : Set of regenerative states
 SU_0/D_0 : Original / Duplicate Unit in normal mode and operative
 $\overline{SU_0}/\overline{D_0}$: Original / Duplicate Unit in normal mode but not working
 Dcs : Duplicate Unit in normal mode and in cold standby
 a/b : Probability that repair is feasible / not feasible
 λ_1/λ_2 : Constant failure rate of Original / duplicate unit.
 $M_i(t)$: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state.
 $W_i(t)$: Probability that the server is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states.
 $q_{ij}(t)/Q_{ij}(t)$: pdf / cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state without visiting any other regenerative state in $(0,t]$.
 $q_{ij,kr}(t)/Q_{ij,kr}(t)$: pdf / cdf of first passage time from a regenerative state i to regenerative state j or to a failed state j visiting states k, r once in $(0,t]$.
 $p_{ij}/p_{ij,kr}$: Probability of transition from regenerative state i to a regenerative state j without visiting any other state in $(0,t]$ / visiting state k, r once in $(0,t]$ that is,
 $p_{ij} = \lim_{s \rightarrow 0} q_{ij}^*(s)$ and $p_{ij,kr} = \lim_{s \rightarrow 0} q_{ij,kr}^*(s)$.
 $h(t)/H(t)$: pdf / cdf of inspection time
 $g_1(t)/G_1(t)$: pdf / cdf of repair time of the server for original unit.
 $g_2(t)/G_2(t)$: pdf / cdf of repair time of the server for the duplicate unit.
 $S(F_{wi}/F_{wi}/F_{ui}/F_{ui})$: Original (Unit is failed and waiting for inspection / waiting for inspection continuously from previous state/ under inspection / under inspection continuously from previous state)
 SF_{ur}/SF_{UR} : Original unit is completely failed and under repair / under repair continuously from previous state.
 DF_{ur}/DF_{UR} : Duplicate unit is failed and under repair / under repair continuously from previous state.
 DF_{wr}/DF_{wR} : Duplicate unit is failed and waiting for repair / waiting for repair continuously from previous state.

$\phi_i(t)$: cdf of first passage time from regenerative state i to a failed state.
 $A_i(t)$: Probability that the system is in up-state at instant t given that the system entered regenerative state i at $t=0$
 $B_i(t)$: Probability that the server is busy at an instant time t given that the system entered the regenerative state i at $t=0$.
 $N_i(t)$: Expected number of visits by the server in $(0,t]$ given that the system entered the regenerative state i at $t=0$.
 \otimes/\odot : Symbols for Stieltjes convolution / Laplace convolution
 $\sim/^{*}$: Symbols for Laplace Stieltjes transform (LST)/Laplace transforms (LT).
 $'$: Symbol for derivative of the function.

Using these notations, the possible transition between states along with transition rates for model I is shown in Figure 1. The transition diagram for model II is same as that of model I except state S_{10} . In model II, $S_{10} = (\overline{SU_0}, SF_{ui}, DF_{w_r})$ and also there are transitions from S_{10} to S_8 and S_{10} to S_9 with probabilities $ah(t)$ and $bh(t)$ respectively.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements;

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \quad (3.1)$$

For model I

$$p_{01}=1, \quad p_{10}=bh^*(\lambda_1+\lambda_2), \quad p_{12}=ah^*(\lambda_1+\lambda_2), \quad p_{14}=\frac{\lambda_2}{\lambda_1+\lambda_2} [1-h^*(\lambda_1+\lambda_2)], \quad (3.2)$$

$$p_{15}=\frac{\lambda_1}{\lambda_1+\lambda_2} [1-h^*(\lambda_1+\lambda_2)], \quad p_{20}=g_1^*(\lambda_1+\lambda_2), \quad p_{23}=\frac{\lambda_1}{\lambda_1+\lambda_2} [1-g_1^*(\lambda_1+\lambda_2)],$$

$$p_{27}=\frac{\lambda_2}{\lambda_1+\lambda_2} [1-g_1^*(\lambda_1+\lambda_2)], \quad p_{48}=a = p_{56} \quad p_{49}=b =$$

$$p_{51}, \quad p_{61}=p_{79}=p_{89}=1 = p_{31}=p_{101} \\ p_{90}=g_2^*(2\lambda_1), \quad p_{910}=1-g_2^*(2\lambda_1)$$

clearly,

$$p_{01}=p_{10}+p_{12}+p_{14}+p_{15}=p_{20}+p_{23}+p_{27}=p_{31}=p_{48}+p_{49}=p_{51}+p_{56}=p_{79} \\ =p_{89}=p_{90}+p_{910}=p_{101}=1$$

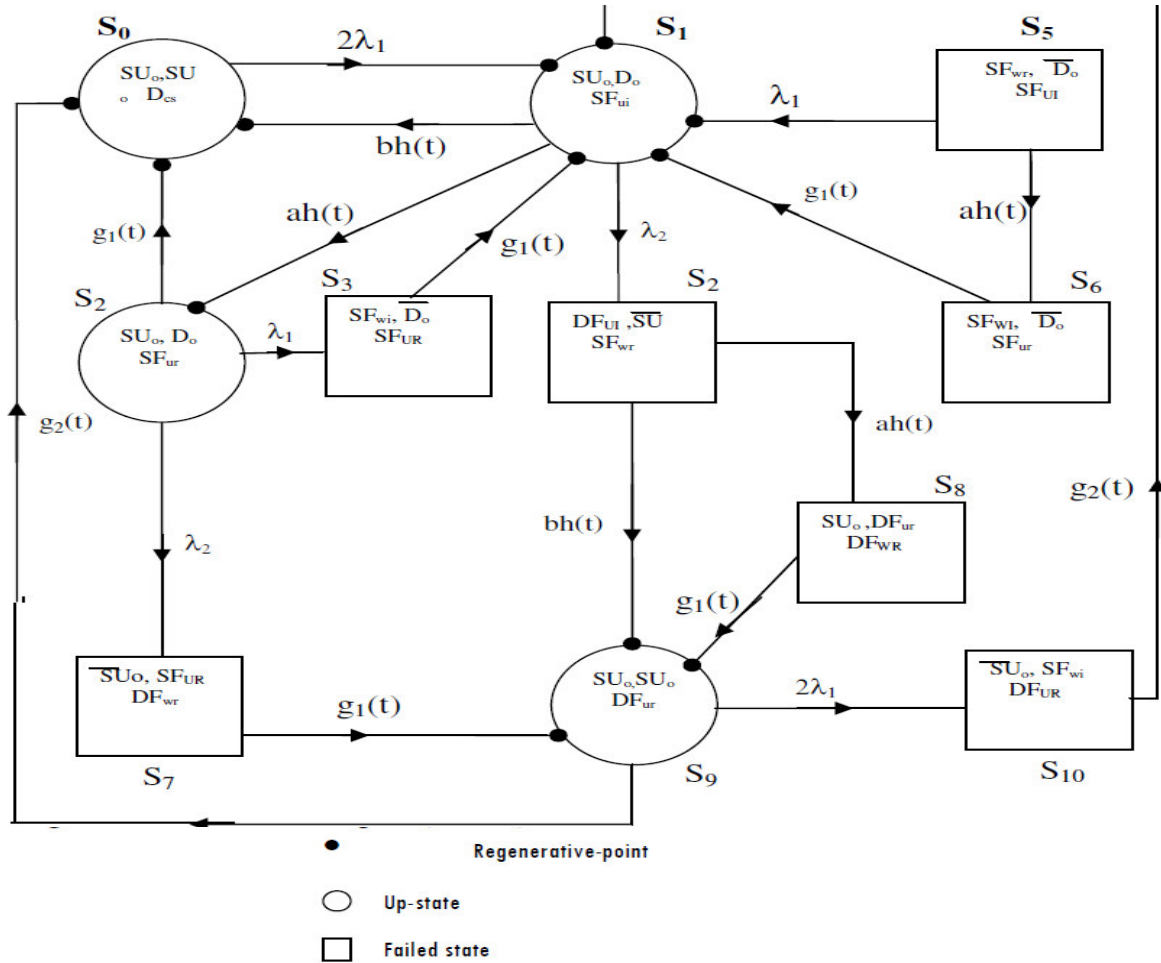


Figure 1. State transition diagram (Mode 1).

Furthermore,

$$p_{11.5} = \frac{b\lambda_1}{\lambda_1 + \lambda_2} [1 - h^*(\lambda_1 + \lambda_2)], \quad p_{11.56} = \frac{a\lambda_1}{\lambda_1 + \lambda_2} \quad [1-$$

$$p_{19.4} = \frac{b\lambda_1}{\lambda_1 + \lambda_2} [1 - h^*(\lambda_1 + \lambda_2)], \quad p_{19.48} = \frac{a\lambda_1}{\lambda_1 + \lambda_2} \quad [1-$$

$$p_{21.3} = \frac{\lambda_1}{\lambda_1 + \lambda_2} [1 - g_1^*(\lambda_1 + \lambda_2)] = p_{23},$$

$$p_{29.7} = \frac{\lambda_2}{\lambda_1 + \lambda_2} [1 - g_1^*(\lambda_1 + \lambda_2)] = p_{27}, \quad p_{91.10} = 1 - g_2^*(2\lambda_1) = p_{910}$$

For model II

The transition probabilities for model II are same as that of model I while the remaining are $p_{108}=a, p_{109}=b, p_{109.8}=b$, clearly, $p_{108} + p_{109} = p_{109} + p_{109.8} = 1$

The unconditional mean time taken by the system to transit to any regenerative state S_j when it (time) is counted from epoch of entrance into that state S_i is given by;

$$m_{ij} = \int_0^\infty t d\{Q_{ij}(t)\} = -q^{*'}_{ij}(0) \quad (3.4)$$

and the mean sojourn time in the state S_i is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t) dt \quad (3.5)$$

where T denotes the time to system failure. Using these, the following results can be obtained:

For model I

$\mu_0 = m_{01}, \mu_1 = m_{10} + m_{12} + m_{14} + m_{15}, \mu_2 = m_{20} + m_{23} + m_{27}, \mu_3 = m_{31}, \mu_4 = m_{48} + m_{49}, \mu_5 = m_{51} + m_{56}, \mu_6 = m_{61}, \mu_7 = m_{79}, \mu_8 = m_{89}, \mu_9 = m_{90} + m_{910}, \mu_{10} = m_{101}.$

For model II

The expressions for μ_i ($i=0,1,\dots,9$) are same as defined in model I except

$$\mu_{10} = m_{108} + m_{109}$$

ANALYSES FOR MODEL I

Mean time to system failure (MTSF)

On the basis of arguments used for regenerative processes, we obtain the following recursive relations for $\phi_i(t)$;

$$\begin{aligned} \phi_0(t) &= Q_{01}(t)\phi_1(t) \\ \phi_1(t) &= Q_{10}(t) \phi_0(t) + Q_{11}(t) \phi_1(t) + Q_{12}(t) \phi_2(t) \\ \phi_2(t) &= Q_{20}(t) \phi_0(t) + [Q_{23}(t) \phi_2(t) + Q_{27}(t)] \end{aligned} \tag{4.1}$$

Taking LST of above relations (4.1) and solving for $\tilde{\phi}_0(s)$, we get MTSF as;

$$\begin{aligned} \text{MTSF } (T_1) &= \lim_{s \rightarrow 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_{11}}{D_{11}}, \quad \text{where,} \\ N_{11} &= (\lambda_1 + \lambda_2) t 2\lambda_1 [1 - h^*(\lambda_1 + \lambda_2) \{1 + a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}] \\ D_{11} &= (\lambda_1 + \lambda_2) 2\lambda_1 [1 - h^*(\lambda_1 + \lambda_2) \{b + a g_1^*(\lambda_1 + \lambda_2)\}] \end{aligned} \tag{4.2}$$

Steady state availability

The recursive relations for $A_i(t)$ are given as

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) \\ A_1(t) &= M_1(t) + q_{10}(t) \odot A_0(t) + [q_{11.5}(t) + q_{11.56}(t)] \odot A_1(t) + q_{12}(t) \odot A_2(t) \\ &\quad + [q_{19.4}(t) + q_{19.48}(t)] \odot A_9(t) \\ A_2(t) &= M_2(t) + q_{20}(t) \odot A_0(t) + q_{21.3}(t) \odot A_1(t) + q_{29.7}(t) \odot A_9(t) \\ A_9(t) &= M_9(t) + q_{90}(t) \odot A_0(t) + q_{91.10}(t) \odot A_1(t), \text{ where,} \\ M_0(t) &= e^{-2\lambda_1 t}, M_1(t) = H(t) e^{-(\lambda_1 + \lambda_2)t}, M_2(t) = G_1(t) e^{-(\lambda_1 + \lambda_2)t}, \\ M_9(t) &= G_2(t) e^{-2\lambda_1 t} \end{aligned} \tag{4.4}$$

Taking LT of above relations (4.4) and solving for $A_0^*(s)$, we get steady state availability of the system as

$$\begin{aligned} A_{10} &= \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_{12}}{D_{12}}, \quad \text{where,} \\ N_{12} &= (\lambda_1 + \lambda_2) [\{b + a g_1^*(\lambda_1 + \lambda_2)\} h^*(\lambda_1 + \lambda_2) + 1] + g_2^*(2\lambda_1) [\lambda_2 - \{ \lambda_2 - a \{1 - g_1^*(\lambda_1 + \lambda_2)\} \} \\ &\quad h^*(\lambda_1 + \lambda_2)] + a h^*(\lambda_1 + \lambda_2) \{1 - g_1^*(\lambda_1 + \lambda_2) + \lambda_2 [1 - g_2^*(2\lambda_1)] [1 - a \{1 - g_1^*(\lambda_1 + \lambda_2)\} \} \} \end{aligned} \tag{4.5}$$

$$\begin{aligned} D_{12} &= (\lambda_1 + \lambda_2) [\{b + a g_1^*(\lambda_1 + \lambda_2)\} h^*(\lambda_1 + \lambda_2) + 1] + g_2^*(2\lambda_1) [\lambda_2 - \{ \lambda_2 - a \{1 - g_1^*(\lambda_1 + \lambda_2)\} \} \\ &\quad h^*(\lambda_1 + \lambda_2)] + a h^*(\lambda_1 + \lambda_2) \{1 - g_1^*(\lambda_1 + \lambda_2) + \lambda_2 [1 - g_2^*(2\lambda_1)] [1 - a \{1 - g_1^*(\lambda_1 + \lambda_2)\} \} \} \\ &\quad - (2\lambda_1) [[a h^*(\lambda_1 + \lambda_2) \{1 - g_1^*(\lambda_1 + \lambda_2)\} g_1^*(0) - \{h^*(0) + a g_1^*(0)\} \{1 - h^*(\lambda_1 + \lambda_2)\}] (\lambda_1 + \lambda_2) - g_2^*(0) \{1 - g_2^*(2\lambda_1)\} \\ &\quad [\lambda_2 \{1 - h^*(\lambda_1 + \lambda_2)\} \{1 - a h^*(\lambda_1 + \lambda_2)\} \{1 - g_1^*(\lambda_1 + \lambda_2)\} \}] \end{aligned} \tag{4.6}$$

Busy period of the server

The recursive relations for $B_i(t)$ are given as

$$\begin{aligned} B_0(t) &= q_{01}(t) \odot B_1(t) \\ B_1(t) &= W_1(t) + q_{10}(t) \odot B_0(t) + [q_{11.5}(t) + q_{11.56}(t)] \odot B_1(t) + q_{12}(t) \odot B_2(t) \\ &\quad + [q_{19.4}(t) + q_{19.48}(t)] \odot B_9(t) \\ B_2(t) &= W_2(t) + q_{20}(t) \odot B_0(t) + q_{21.3}(t) \odot B_1(t) + q_{29.7}(t) \odot B_9(t) \\ B_9(t) &= W_9(t) + q_{90}(t) \odot B_0(t) + q_{91.10}(t) \odot B_9(t), \text{ where,} \\ W_1(t) &= [e^{-(\lambda_1 + \lambda_2)t} + \{ \lambda_1 e^{-(\lambda_1 + \lambda_2)t} \odot 1 \} + \{ \lambda_2 e^{-(\lambda_1 + \lambda_2)t} \odot 1 \} H(t) + \\ &\quad [\lambda_1 e^{-(\lambda_1 + \lambda_2)t} \odot a h(t) \odot 1 \} + \{ \lambda_2 e^{-(\lambda_1 + \lambda_2)t} \odot a h(t) \odot 1 \} G_1(t)] \\ W_2(t) &= [e^{-(\lambda_1 + \lambda_2)t} + \{ \lambda_1 e^{-(\lambda_1 + \lambda_2)t} \odot 1 \} + \{ \lambda_2 e^{-(\lambda_1 + \lambda_2)t} \odot 1 \} G_1(t)] \\ W_9(t) &= [e^{-2\lambda_1 t} + (2\lambda_1^{-2\lambda_1 t} \odot 1) G_2(t)] \end{aligned} \tag{4.7}$$

Taking LT of relations (4.7) and solving for $B_0^*(s)$, we get in long run the time for which the system is under repair/inspection as

$$B_{10} = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_{13}}{D_{12}}, \tag{4.8}$$

where

$$N_{13} = \{ [W_1^*(0) + a h^*(\lambda_1 + \lambda_2) W_2^*(0)] (\lambda_1 + \lambda_2) + \lambda_2 [1 h^*(\lambda_1 + \lambda_2) \{1 - a \{1 - g_1^*(\lambda_1 + \lambda_2)\} \} W_9^*(0) (2\lambda_1)] \}$$

and D_{12} is already specified

Expected number of visits by the server

The recursive relations for $N_i(t)$ are given as

$$\begin{aligned} N_0(t) &= Q_{01}(t) [1 + N_0(t)] \\ N_1(t) &= Q_{10}(t) N_0(t) + [Q_{11.5}(t) + Q_{11.56}(t)] N_1(t) + Q_{12}(t) N_2(t) \\ &\quad + [Q_{19.4}(t) + Q_{19.48}(t)] N_9(t) \\ N_2(t) &= Q_{20}(t) N_0(t) + Q_{21.3}(t) N_1(t) + Q_{29.7}(t) N_9(t) \\ N_9(t) &= Q_{90}(t) N_0(t) + Q_{91.10}(t) N_9(t) \end{aligned} \tag{4.10}$$

Taking LST of above relations (4.10) and solving for $\tilde{N}_0(s)$, we get the expected number of visits per unit time as;

$$N_{10} = \lim_{s \rightarrow 0} s N_0^*(s) = \frac{N_{14}}{D_{12}} \tag{4.11}$$

where

$$N_{14} = (2\lambda_1)g_2^*(2\lambda_1)[(\lambda_2 - \{\lambda_2 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\})h^*(\lambda_1 + \lambda_2)] \quad (4.12)$$

and D_{12} is already specified.

ANALYSES FOR MODEL II

Mean time to system failure (MTSF)

MTSF of model II is same as that of model I.

Steady state availability

The recursive relations for $A_i(t)$ ($i=0,1,2$) are same as defined in case of model I except the following expressions:

$$A_9(t) = M_9(t) + q_{90}(t) \odot A_0(t) + q_{910}(t) \odot A_{10}(t) \quad A_{10}(t) = [q_{109}(t) + q_{109.8}(t)] \odot A_9(t)$$

Where,

$$M_0(t) = e^{-2\lambda_1 t}, \quad M_1(t) = H(t)e^{-\tau(\lambda_1 + \lambda_2)t}, \quad M_2(t) = G_1(t) e^{-(\lambda_1 + \lambda_2)t},$$

$$M_9(t) = G_2(t)e^{-2\lambda_1 t}$$

Proceeding similarly as in model I, we get steady state availability of the system as

$$A_{20} = \frac{N_{22}}{D_{22}} \quad (5.2)$$

where,

$$N_{22} = [(\lambda_1 + \lambda_2) - \lambda_1 [1 - h^*(\lambda_1 + \lambda_2)\{1 + a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}] + (2\lambda_1)[1 - h^*(\lambda_1 + \lambda_2)\{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}]]g_2^*(2\lambda_1) + \lambda_2[1 - h^*(\lambda_1 + \lambda_2)\{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}] \{1 - g_2^*(2\lambda_1)\} \text{ and}$$

$$D_{22} = [(\lambda_1 + \lambda_2) - \lambda_1 [1 - h^*(\lambda_1 + \lambda_2)\{1 + a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}] + (2\lambda_1)[1 - h^*(\lambda_1 + \lambda_2)\{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}]]g_2^*(2\lambda_1) + \lambda_2[1 - h^*(\lambda_1 + \lambda_2)\{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}] \{1 - g_2^*(2\lambda_1)\} - (2\lambda_1)[g_2^*(2\lambda_1)\{g_1^{*(0)}(a\lambda_1) h^*(\lambda_1 + \lambda_2)\{1 - g_1^*(\lambda_1 + \lambda_2)\} + \lambda_2 h^{*(0)}\{1 - h^*(\lambda_1 + \lambda_2)\}\} + \{g_1^{*(0)} + a h^{*(0)}\}\{1 - h^*(\lambda_1 + \lambda_2)\} + a\lambda_2 g_1^{*(0)}\{1 - g_1^*(\lambda_1 + \lambda_2)\} h^*(\lambda_1 + \lambda_2)\} + \lambda_2 h^{*(0)}\{1 - g_2^*(2\lambda_1)\}\{1 - h^*(\lambda_1 + \lambda_2)\} \{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\} + a\lambda_2 g_1^{*(0)}\{1 - g_2^*(2\lambda_1)\}\{1 - h^*(\lambda_1 + \lambda_2)\} \{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\} + g_2^*(2\lambda_1)\{1 - h^*(\lambda_1 + \lambda_2)\}]] \quad (5.3)$$

Busy period of the server

The recursive relations for $B_i(t)$ ($i=0,1,2$) and W_i ($i=1,2$) are same as defined in case of model I except the following expressions:

$$B_9(t) = W_9(t) + q_{90}(t) \odot B_0(t) + q_{910}(t) \odot B_{10}(t)$$

$$B_{10}(t) = W_{10}(t) + [q_{109}(t) + q_{109.8}(t)] \odot B_9(t)$$

where,

$$W_9(t) = H(t) + (\overline{ah}(t) \odot 1)G_1(t)$$

Proceeding similarly as in model I, we get in the long run the time for which the system is under repair as;

$$B_{20} = \frac{N_{23}}{D_{22}}, \quad (5.6)$$

where

$$N_{23} = (2\lambda_1)[(\lambda_1 + \lambda_2)g_2^*(2\lambda_1)\{W_1^*(0) + aW_2^*(0)h^*(\lambda_1 + \lambda_2)\} + \lambda_2 [1 - h^*(\lambda_1 + \lambda_2)\{1 - a\{1 - g_1^*(\lambda_1 + \lambda_2)\}\}]] [W_9^*(0) + \{1g_2^*(2\lambda_1)\}W_{10}^*(0)] \quad (5.7)$$

and D_{22} is already specified

Expected number of visits by the server

The recursive relations for $N_i(t)$ ($i=0,1,2$) are same as defined in case of model I except the following expressions:

$$N_9(t) = Q_{90}(t) \quad N_0(t) \odot Q_{910}(t) \quad N_9(t) \odot N_{10}(t) = [Q_{109}(t) + Q_{109.8}(t)] \quad N_9(t) \odot N_{10}(t)$$

Proceeding similarly as in model I, we get the expected number of visits per unit time as;

$$N_{20} = \frac{N_{24}}{D_{22}}, \text{ where}$$

$$N_{24} = (2\lambda_1)g_2^*(2\lambda_1)[(\lambda_1 + \lambda_2) - \lambda_1\{1 - h^*(\lambda_1 + \lambda_2)\}\{1 - a\{1g_1^*(\lambda_1 + \lambda_2)\}\}]] \quad (5.10)$$

and D_{22} is already specified.

PROFIT ANALYSIS

Profit incurred to the system models in steady state are given by

$$P_1 = K_1A_{10} - K_2B_{10} - K_3N_{10} \quad (\text{for model I}) \quad (6.1)$$

$$P_2 = K_1A_{20} - K_2B_{20} - K_3N_{20} \quad (\text{for model II}) \quad (6.2)$$

where

K_1 = Revenue per unit up time of the system

K_2 = Cost per unit time for which server is busy

K_3 = Cost per unit visit by the server.

Particular case

Let us take $g_1(t) = \theta_1 e^{-\theta_1 t}$, $g_2(t) = \theta_2 e^{-\theta_2 t}$ and $h(t) = \alpha e^{-\alpha t}$, we can obtain the following results;

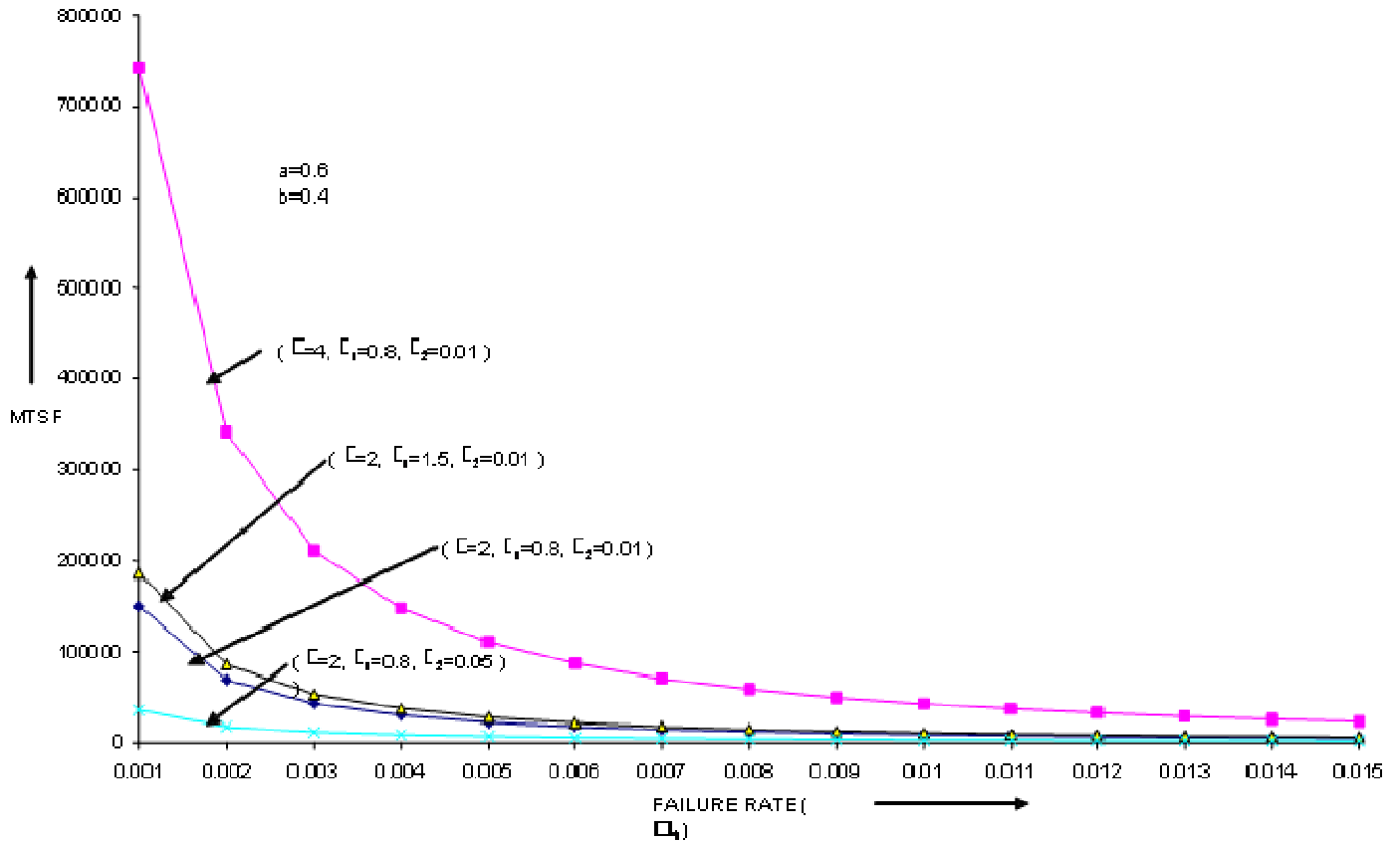


Figure 2. MTSF vs. failure rate.

For Model I

$$N_{11} = (\theta_1 + \lambda_1 + \lambda_2)(\alpha + 3\lambda_1 + \lambda_2) + (2\lambda_1 a \alpha) \quad (7.1)$$

$$D_{11} = (2\lambda_1) [(\theta_1 + \lambda_1 + \lambda_2)(\alpha + \lambda_1 + \lambda_2 - b\alpha) - (a\alpha\theta_1)]$$

$$N_{12} = [(\theta_1 + \lambda_1 + \lambda_2)\{(\alpha + \lambda_2)(\theta_2 + 2\lambda_1) - (2\lambda_1\lambda_2)\} + (a\alpha\lambda_1)(\theta_2 + 2\lambda_1 + 2\lambda_2) + 2\lambda_1(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)(\theta_2 + 2\lambda_1 + \lambda_2)] \theta_1 \theta_2 \alpha \quad (7.2)$$

$$N_{13} = (\theta_1 + \lambda_1 + \lambda_2 + a\alpha)(\theta_2 + 2\lambda_1 + 2\lambda_2)(2\lambda_1\theta_1\theta_2\alpha) \quad (7.2)$$

$$N_{14} = \frac{[(\alpha + \lambda_2)(\theta_1 + \lambda_1 + \lambda_2) - (a\alpha\lambda_1)](\theta_2 + 2\lambda_1) - 2\lambda_1\lambda_2}{(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)(2\lambda_1\alpha\theta_1\theta_2)} \quad (7.3)$$

$$D_{12} = \{(\alpha + \lambda_2)(\theta_1 + \lambda_1 + \lambda_2) - a\alpha\lambda_1 + 2\lambda_1(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)\}(\theta_2 + 2\lambda_1)(\alpha\theta_1\theta_2) + (\lambda_1 + \lambda_2)(2\lambda_1\theta_2)(\theta_2 + 2\lambda_1)\{a\alpha^2 + (\theta_1 + a\alpha)(\theta_1 + \lambda_1 + \lambda_2)\} + 4\lambda_1^2\lambda_2\alpha\theta_1(\theta_1 + \lambda_1 + \lambda_2 + a\alpha) \quad (7.4)$$

For Model II

$$N_{22} = \frac{[\theta_2\{(\theta_1 + \lambda_1 + \lambda_2)(\alpha + \lambda_2) + a\alpha\lambda_1 + (\alpha + \lambda_1 + \lambda_2)(\theta_1 + \lambda_1 + \lambda_2)\} + 2\lambda_1\lambda_2(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)(\alpha\theta_1)]}{(7.5)}$$

$$N_{23} = [\theta_2 + \{(\theta_2 + 2\lambda_1)\lambda_2\}(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)](2\lambda_1\alpha\theta_1) \quad (7.6)$$

$$N_{24} = (2\lambda_1\alpha\theta_1\theta_2)[(\alpha + \lambda_1 + \lambda_2)(\theta_1 + \lambda_1 + \lambda_2) - \lambda_1(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)] \quad (7.7)$$

$$D_{22} = \frac{\alpha\theta_1\theta_2\{(\theta_1 + \lambda_1 + \lambda_2)(\alpha + \lambda_2) + a\alpha\lambda_1 + (\alpha + \lambda_1 + \lambda_2)(\theta_1 + \lambda_1 + \lambda_2)\} + 2\theta_1\lambda_1\lambda_2\theta_2(\theta_1 + \lambda_1 + \lambda_2 + a\alpha) + \{(\alpha^2\lambda_1) + \theta_1 + \lambda_2(\theta_1 + \lambda_1 + \lambda_2)\}(\theta_1 + \lambda_1 + \lambda_2 + a\alpha)}{2\lambda_1\theta_2 + (\theta_1 + a)(2\lambda_1^2\theta_2)(\theta_1 + \lambda_1 + \lambda_2) + 2\lambda_1\lambda_2\alpha\{(2\lambda_1)(\theta_1 + \lambda_1 + \lambda_2 + a\alpha) + \theta_2(\theta_1 + \lambda_1 + \lambda_2)\} + 2\lambda_1\lambda_2\{\alpha^2\theta_2 + (\theta_1 + \lambda_1 + \lambda_2 + a\alpha)\}} \quad (7.8)$$

Conclusion

The mean time to system failure (MTSF) of the system models is same which decreases with the increase of failure rates of the units as shown in Figure 2. However, it increases when inspection and repair rates increase. The behaviour of the profit difference ($P_2 - P_1$) of the system models is shown in the Figure 3. This figure indicates that the profit difference goes on increasing as failure rates increase with fixed values of other parameters. But this difference becomes less with the increase of repair and

PROFIT DIFFERENCE V/S FAILURE RATE

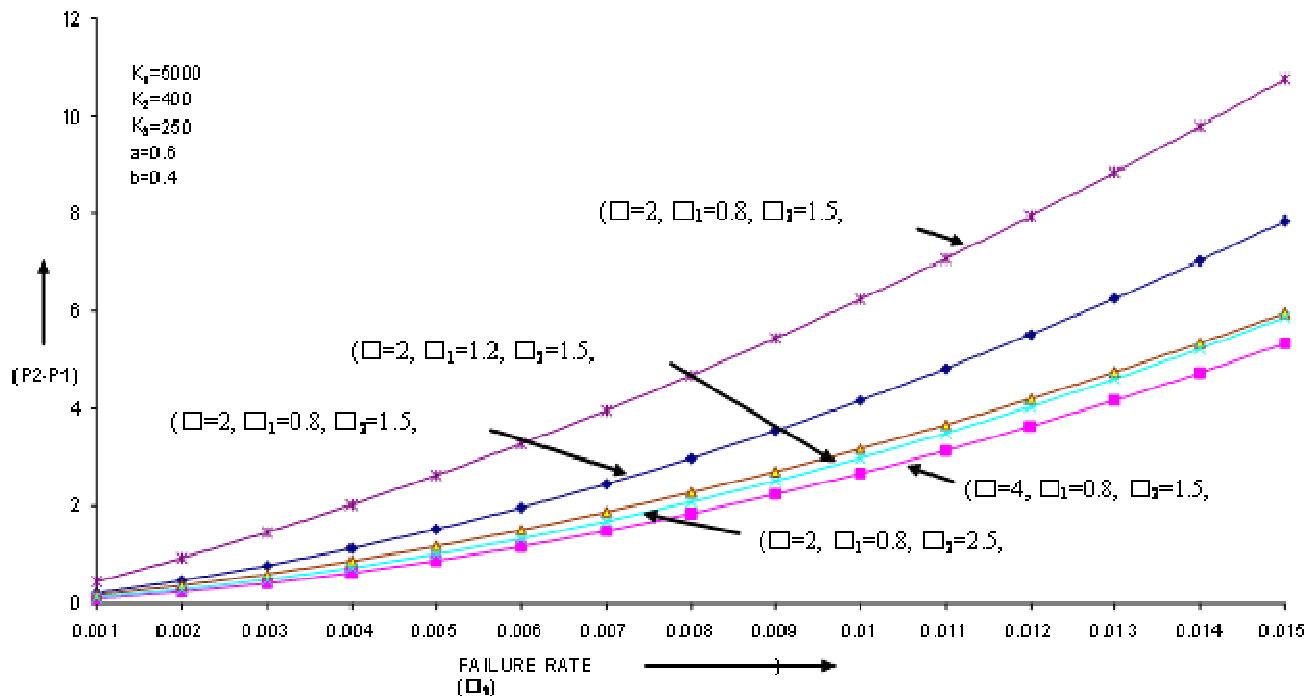


Figure 3. Profit difference vs. failure rate

inspection rates. Thus on the basis of results obtained for a particular case, it is concluded that a 2-out-of-3 redundant system having one duplicate unit can be made always profitable by giving priority to repair the original unit over the duplicate unit and by replacing the original unit at its failure by new one in case chances of feasibility of repair is less.

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