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Full Length Research Paper

Analysis of risk in linear multi-objective model and its evaluation for selection of a portfolio of investment in the Mexican Stock Exchange

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Each model designed to select an investment portfolio is based on different assumptions for estimating the risk and the return. These assumptions determine feasible solutions area and the front of efficient portfolios. Therefore, the assumptions of Markowitz model, capital assets pricing model and the linear multi-objective model are discussed. A portfolio of investments in different scenarios was determined by the last two models, and it is showed that the portfolio of investment determined by linear multi-objective model has higher return at lower risk. These tests also evaluated the assumptions of the models. It concludes that it is possible to gain a competitive advantage if another point of view will be considered in the selection of the investment portfolio.

Key words: Multi-objective linear programming, investment portfolio, Mexican Stock Exchange.

INTRODUCTION

In a portfolio of investment, the assessment of the profitability and risk expected enclose difficulties, because both one and another are linked to future situations not known with certainty. Therefore, models are needed to better understand these concepts and try to benefit from this improved knowledge.

Financial risks are related to the possibility of losses generated in financial activities. Some of them are unfavourable interest rates and movements of stock prices change. Financial institutions do not seek to eliminate the risks, this is impossible, they seek to manage and control them. It is therefore, necessary to identify them and measure them. Risks are taken, and in the worst case, occur when a risk profile is not the product of a calculated decision, as it is an operational market seeking profitability or growth, forgetting the risk (Gonzalez and Garcia, 2008).

The initial assumption of scanned models is that they have a sum of money to invest in the present. This money will be invested for a period of time, known as the period of tenure. The values that were purchased will be sold at the end of the period of tenure and their benefits will be used for expenses, reinvestment or both. At this point, the models will be applied again to benefits, to reinvest. At t = 0, it will be decided what values to buy and keep until t = 1. Then, at t = 1, it will be decided what new values to buy and keep up to t = 2, and so forth. Maximize the expected performance and minimize risk are two objective functions in conflict, consequently, it is necessary to assess the decision to purchase (Gonzalez and Garcia, 2008). Each model solves these functions with different assumptions and procedures.

In this paper, the assumptions and the procedures for the calculation of the risk of the investment portfolio are analysed, because the selection of the investment portfolio is determined by these processes using their assumptions. In the three models analysed, Markowitz model (Markowitz, 1952), the capital assets pricing model (CAPM) (Fama and French, 2003) and linear multi-objective model (LMM) (Zavala-Diaz et al., 2010), the risk of the portfolio is determined differently, though all of them are based on statistic.

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The paper is organized as describing the assumptions of Markowitz, CAPM and the LMM; two Mexican Stock Market (Bolsa Mexicana de Valores, (BMV)) investment portfolios are selected, where the number of titles is small and it is useful for showing the operation of the CAPM and the LMM. Later, the result is discussed and, finally, the conclusion is presented.

THE CALCULATION OF THE RISK BASE ON INVESTMENT PORTFOLIO

Two models are essential to the evaluation of an investment portfolio, Markowitz (1952) and the CAPM (Fama and French, 2003). But they are not unique because, multi-objective models have been developed for selecting investment portfolio (Fieldsend and Singh, 2002; Subbu et al., 2005; Branke et al., 2009; Zavala-Diaz et al., 2010). In these multi-objective models, the efficient portfolios border is the front of Pareto, where non-dominated or optimal solutions are. To determine these solutions, let us assume that the search space is convex (Coello et al., 2002), and the route of this space is performed by means of evolutionary algorithms (Fieldsend and Singh, 2002; Subbu et al., 2005; Branke et al., 2009), or by means of procedures based on the methods of solving problems of linear programming, as our model developed recently (Zavala-Diaz et al., 2010). In some multi-objective approaches, the risk and the return are not always calculated as in Markowitz model or the CAPM (Fieldsend and Singh, 2002; Deng et al., 2010a, b; Zavala-Diaz et al., 2010), and others use additional restrictions as the price of sale (Fieldsend and Singh, 2002; Zavala-Diaz et al., 2010).

Reference models

In the Markowitz model, a portfolio will be efficient if it has the least possible risk to a certain level of profitability. The set of efficient portfolios is estimated solving the quadratic parametric problem (Markowitz, 1952):

$$\min \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} x_i R_i = V^* \quad (2)$$

$$\sum_{i=1}^{n} x_i = 1, 0 \le x_i \le 1 \quad (i = 1, 2, ..., n) \quad (3)$$

Where:

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \quad (4)$$

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{m}\left(r_{i_j} - R_i\right)^2}{m-1}} \quad (5)$$

$$R_i = \frac{\sum_{j=1}^{m} r_{i_j}}{m} \quad (6)$$

$$\rho_{12} = \frac{\sum_{j=1}^{m} r_{1_j} r_{2_j} - \frac{\left(\sum_{j=1}^{m} r_{1_j}\right)\left(\sum_{j=1}^{m} r_{2_j}\right)}{m}}{\sqrt{\sum_{j=1}^{m}\left(r_{1_j} - R_1\right)^2 \sum_{j=1}^{m}\left(r_{2_j} - R_2\right)^2}} \quad (7)$$

x_i is the unknown of the problem and the proportion of financial asset i, σ_p^2 is the variance portfolio p, σ_{ij} is the covariance between the returns of the actions x_i and x_j. ρ_{ij} is the correlation between the returns of the actions x_i and x_j. R_p expected performance portfolio p and is equal to V^*. V^* is a parameter to vary gets the set of proportions x_i while minimizing the risk of the portfolio. R_i is the average performance of the action i, r_i is the performance of the i in every j period action, m is the number of considered periods and n is the number of titles.

The risk and the return for each asset are determined by the Equations (6) and (5), respectively. The results of this model is the set of combinations return-risk [R_p, σ_p^2], whose limit is given by efficient portfolios that are in the "efficient portfolios frontier".

The CAPM which is based on any financial asset can be described in two statisticians (Fama and French, 2003): in position, the media provides a measure of the average return on the asset in a given period; the dispersion, the standard deviation of different titles average performance, provides a measure of the risk of financial assets (Branke et al., 2009). In this model, the return and the risk of the investment portfolio are determined with the following equations:

$$\text{Return}: R_p = \sum_{i=1}^{n} x_i R_i \quad (8)$$

$$\text{Risk}: \sigma_p = \sqrt{\sum_{i=1}^{n}\left(x_i^2\sigma_i^2 + x_j^2\sigma_j^2\right) + 2\sum_{i=1}^{n}\left(x_i x_j \sigma_i \sigma_j \rho_{ij}\right)} \quad (9)$$

Equation (8) is the same as the Markowitz model Equation (2).

Investment portfolio is formed by assigning investment percentages to financial assets x_i in the Equations (8) and (9). The resulting portfolio as well as individual actions will be identified by the average return and their associated risk.

The condition to comply is the sum of these percentages is equal to 100%, similar to one of the constraints of the model of Markowitz, Equation (3) condition. The investment portfolio selected corresponds to portfolio that has the smallest difference between risk and profit.

In both models, it is clear that the objective function of maximizes the return is not explicitly calculated. The first model gets the lowest risk for a given return. The second model gets the portfolio with the smallest difference between the risk and the return. In both models, a big number of portfolios are calculated to traverse all feasible solutions space until it reaches the border of efficient portfolios. In both models, the risk is estimated considering the correlation between returns from shares. Hence, it is likely that the risk is composed with the highest percentage of assets that have a low

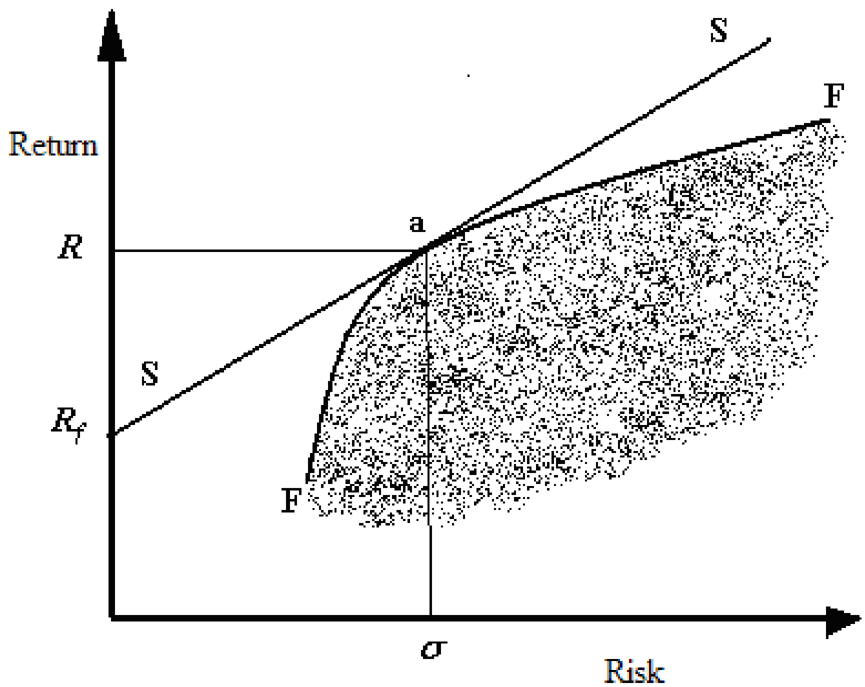

Figure 1. Graphical interaction models of Markowitz and the CAPM (Fama and French, 2003).

With regard to the settlement, the most important difference between the two models is that the Markowitz model determines optimal portfolios that are in the front of efficient portfolios, and in the CAPM its solution is a single portfolio which is in the front of efficient portfolios. Figure 1 shows the relationship between the solutions of Markowitz model with the CAPM.

In Figure 1, the FF line represents the border of efficient portfolios. SS line is the line of security market, where the R_f point represents the risk-free return. Point a on line FF is the portfolio with the smallest difference between risk σ and return R. Consequently, 'a' point is the portfolio determined by the CAPM (Fama and French, 2003).

Basis of linear multi-objective model (LMM)

In the models discussed earlier, the profitability of a portfolio is defined by the weighting of the expected profitability of *n* values that compose it, while the risk is a function of the three factors set out thus (Markowitz, 1952; Fama and French, 2003):

i. The proportion or weighting of each asset in the portfolio (x_i).
ii. Variance or standard deviation of the profitability of each asset (σ_i).
iii. Covariance (σ_{ij}) or the correlation coefficient (ρ_{ij}) between profits of each pair of values.

In the LMM, one of the assumptions is that the portfolio risk is calculated directly with the proportion of standard deviation of each asset (Zavala-Diaz et al., 2010), that is;

$$\sigma_p = \sum_{i=1}^{n} x_i \sigma_i \quad (10)$$

If Equation (10) applies to two assets, x_i and x_j, they will rise to the square and will be the same radical Equation (9). But in order to directly use Equation (10), the correlation coefficient is not considered. In statistical terms, this would mean assets have a strong correlation ($\rho_{ij} \approx 1$).

Approach linear multi objective model (LMM)

The assumptions considered for the preparation of the LMM are (Zavala-Diaz et al., 2010):

1. The minimum required amount of money is the cost of the cheapest action, from this quantity reaches the amount of money of the most expensive action.
2. The minimal risk of the investment portfolio is the risk of the action that may have the minimum of all actions, and it will vary up to the highest risk of the corresponding action. This assumption is based on as described in section of Basis of linear multi-objective model.
3. The minimum profit of the investment portfolio is the return of the action that has lower return of all actions, and it will vary up to the highest return of the corresponding action.
4. The LMM is obtained with the following optimization models (Zavala-Diaz et al., 2009).

Maximization of return

$$\text{Max} \quad z_1 = \sum_{i=1}^{n} x_i R_i \quad (11)$$

Subject to:

$$\sum_{i=1}^{n} x_i \left(Pv_i - Pv_{\min} \right) \leq \lambda_1 \left(Pv_{\max} - Pv_{\min} \right)$$

Table 1. Profit, price and risk of three companies of problem 1.

Var	Company	R_i(%)	Pv_i	σ_i(%)	R_i-R_{min}	σ_i-σ_{min}	σ_i-R_i
X_1	FEMSA	3.845	136.62	4.728	0.783	0.000	0.883
X_2	TELMEX	3.062	17.09	5.863	0.000	1.135	2.801
X_3	G.MODELO	3.250	60.17	6.252	0.188	1.524	3.002

$$\sum_{i=1}^{n} x_i(\sigma_i - \sigma_{\min}) \le \lambda_2(\sigma_{\max} - \sigma_{\min})$$

$$\sum_{i=1}^{n} x_i = 1 \text{, } 0 \le x_i \le 1$$

Minimization of risk

$$\text{Min } z_2 = \sum_{i=1}^{n} x_i \sigma_i \quad (12)$$

Subject to:

$$\sum_{i=1}^{n} x_i (Pv_i - Pv_{\min}) \ge \lambda_1 (Pv_{\max} - Pv_{\min})$$

$$\sum_{i=1}^{n} x_i (R_i - R_{\min}) \ge \lambda_2 (R_{\max} - R_{\min})$$

$$\sum_{i=1}^{n} x_i = 1 \text{, } 0 \le x_i \le 1$$

Where R_i, σ_i and Pv_i is return, risk and price action *i*, respectively. x_i is the ratio of action *i* must purchase and is a real variable $0 \le x_i \le 1$, $x_i = 0$ when the action is not part of the investment portfolio. λ_1, λ_2 are variables that are used to traverse the space solutions and their value is bounded by $0 \le \lambda_1, \lambda_2 \le 1$.

Linear multi objective model (LMM)

The LMM is development with the models (5) and (6), the e-restrictions method and the previously described assumptions. This development includes the process to compute the optimal solution (Zavala-Diaz et al., 2010). The result of the LMM is a single investment portfolio, the optimal portfolio. The LMM approach is:

$$\text{Minimize } z_3 = \sum_{i=1}^{n} x_i (\sigma_i - R_i) \quad (13)$$

Subject to:

$$\sum_{i=1}^{n} x_i(\sigma_i - \sigma_{\min}) = \lambda_1(\sigma_{\max} - \sigma_{\min}) \quad (14)$$

$$\sum_{i=1}^{n} x_i(R_i - R_{\min}) = \lambda_2(R_{\max} - R_{\min}) \quad (15)$$

$$\sum_{i=1}^{n} x_i Pv_i \ge Pv_{\min} \quad (16)$$

$$\sum_{i=1}^{n} x_i = 1 \text{, } 0 \le x_i \le 1$$

Where the variables are the same as those used in optimization base models, except that λ_1 and λ_2, in this model are the parameters used to search for the optimal solution in the space of workable solutions and their value are given by the interval $0 \le \lambda_1, \lambda_2 \le 1$.

Restrictions (14) and (15) are objective functions of two models of optimization basis; their magnitudes will varies from the lower bound until the higher bound, as this is established in the e-restriction method. In addition, these restrictions reflect the second and third assumption of the original approach, and the (16) Restriction reflects the first assumption. The (14), (15) and (16) Restrictions are valid for the whole of the feasible region (Zavala-Diaz et al., 2010). In each iteration of the process of solution of the LMM, a problem of linear programming is resolved. This problem is resolved by the SIMPLEX method (Zavala-Diaz et al., 2010).

RESULTS AND DISCUSSION

The selection of investment portfolios is performed with the CAPM and the LMM. The public information of titles of the BMV was chosen to form two different scenarios. The queried data are from July 2005 to February 2007 (Mexican stock-market 2007).

Problem 1

The profit and risk calculated using the Equations (5) and (6) are tabulated in Table 1. Figure 2a are workable solutions computed with the CAPM assumptions and in Figure 2b, those same solutions are calculated with the LMM assumptions. As shown in Figure 2, the process to calculate the risk will determine the workable solutions space and the border of efficient portfolios. Because the efficient portfolios border is different in each model, this implies that combinations and proportions of assets that are at these borders are also different.

In Figure 2a, it is known that one of the points on theleft will be the investment portfolio with the smallest difference between risk and performance. Instead, despite space workable solutions of Figure 2b, it will have to apply the LMM for obtain optimal investment portfolio. The results are shown in Table 2. In Table 2, in the first line, the results of the CAPM are shown. In the second line are those same results but the risk determined with the LMM assumptions. The third line shows the results obtained by the LMM. Finally, in the fourth line are the

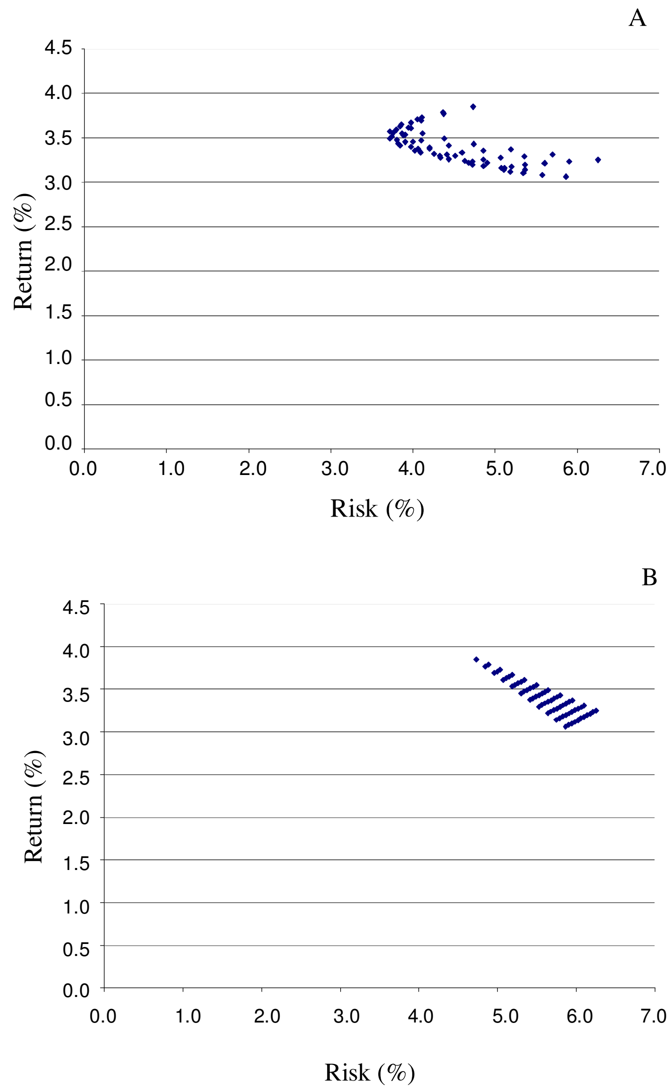

Figure 2. Area of feasible solutions return-risk of problem 1 applying (A) CAPM and (B) LMM.

Table 2. Profit and risk of investment portfolios of problem 1.

Model		X_1	X_2	X_3	Sum	Diff
	Composition	0.6000	0.2000	0.2000	1	
CAPM	Return	2.3075	0.6125	0.6499	3.5699	0.1505
	Risk	--------	-------	-------	3.7204	
	Composition	0.6000	0.2000	0.2000	1	
CAPM risk by LMM	Return	2.3075	0.6125	0.6499	3.5699	1.6902
	Risk	2.8369	1.1727	1.2505	5.2601	
	Composition	0.6171	0.3621	0.0208	1	
LMM	Return	2.3694	1.1080	0.0674	3.5448	1.6198
	Risk	2.9127	2.1219	0.1300	5.1646	
	Composition	0.6171	0.3621	0.0208	1	
LMM risk by CAPM	Return	2.3694	1.1080	0.0674	3.5448	0.3217
	Risk	---------	-------	--------	3.8665	

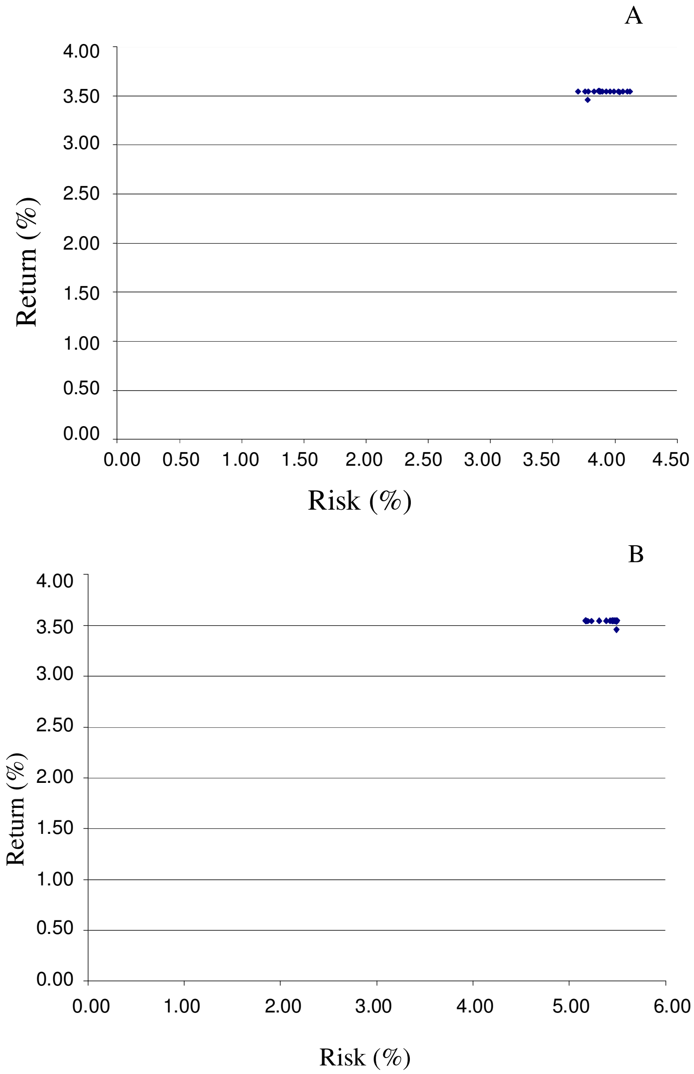

Figure 3. Points to get the optimal portfolio by LMM Space given by (A) CAPM and (B) LMM.

proportions of the three line assets but with risk determined by the CAPM. Also in Table 2, the composition of portfolios determined by both models is not so different, in both portfolios, the highest proportion is the active x_1 and its magnitude in both portfolios is not so different. The difference of the portfolios is the proportion of the other two assets, being these sensitive differences between the portfolios. But in spite of these differences, the profit is similar in the two portfolios; there is a difference in the 0.0251% higher for the CAPM. Risk situation is not so different if one considers the assumptions of the LMM, where the risk of the portfolio determined by CAPM is 0.0955% greater than the portfolio determined by the LMM. On the other hand, if the values obtained with the assumptions of the CAPM are considered, then the risk of the LMM-driven portfolio is greater. But, the important thing is that portfolio determined by the LMM is a portfolio valid into of the feasible solutions of the CAPM.

In Figures 3a and b, the points to determine the optimal portfolio with the LMM are shown. Figure 3a shows the points in the space of workable solutions given by assumptions of the CAPM. Figure 3b shows those same points in the space of workable solutions given by the LMM assumptions.

In these figures, it is observed that the number of points needed to determine the optimal portfolio is smaller than the points required for the CAPM.

Problem 2

In Table 3, the return and the risk of three actions obtained with the Equations (5) and (6) are tabulated. Figure 4a shows the area of feasible solutions of the CAPM and Figure 4b are those same solutions calculated

Table 3. Return, price and risk of three companies issue 2 [11].

Var	Company	R_i(%)	Pv	σ_i(%)	R_i-R_{min}	σ_i-σ_{min}	σ_i-R_i
X_1	WALMART	9.3622	49.04	21.7545	7.2504	16.9207	12.3923
X_2	TELMEX	3.0623	17.09	5.8635	0.9505	1.0297	2.8012
X_3	TVAZTECA	2.1118	9.0	4.8338	0.0000	0.0000	2.7220

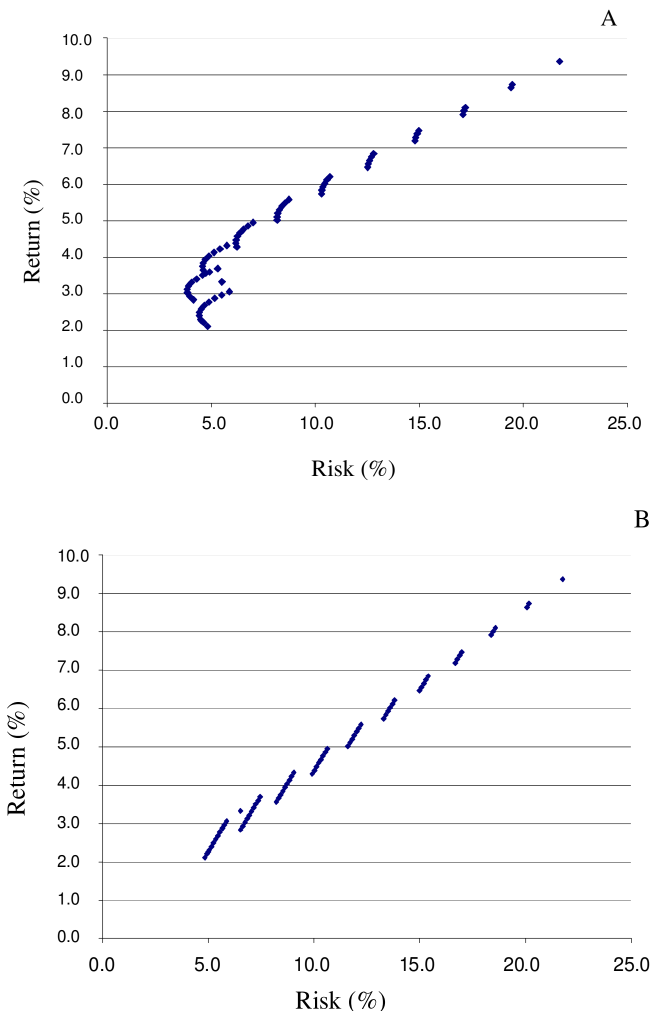

Figure 4. Problem 2 return-risk feasible solution area applying; (A) the CAPM assumptions and (B) the LMM assumptions.

with the LMM assumptions. In these graphs, it is noted that the feasible solutions are similar in the two models. Problem 2 results are displayed in Table 4.

In Table 4, the results are presented in the same order as in Table 2. As shown in this table, the composition of portfolios is different. In the portfolio obtained by the CAPM, the asset x_3 is one that has a larger proportion. While portfolio obtained by the LMM, the asset x_2 has the greater proportion. The proportion of the other two assets is completely different. As a result of these differences the portfolios have different profit and risk. Greater profit is determined by the LMM portfolio, this performance is 0.1074% greater than that determined by the CAPM portfolio performance. The risk will be function of the model concerned, if the risk of both portfolios is calculated with the LMM assumptions, then the selected portfolio by the LMM has a lower risk of 0.6506% compared to the determined by the CAPM. In the case of calculating the risk of both portfolios with the assumptions of the CAPM, the determined portfolio by LMM is a portfolio that is in the region of workable solutions.

The obtained points to determine the optimal portfolio with the LMM are shown in Figures 5a and b. The points in the space of workable solutions given by assumptions of the CAPM are shown in Figure 5a. Those same points in the space of workable solutions given by the LMM assumptions are in Figure 5b. As shown in Figure 5a, the search is in the area around the solution determined by the CAPM. In the graph of Figure 5b, the walk is performed to increase performance.

There is no doubt that in the two problems, the better investment portfolio is determined by LMM, and this portfolio has higher profit at lower risk than the determined by the CAPM. On the other hand, in the first problem the determined investment portfolios have profits and similar risks. The reason of this different behavior in both problems is the cause for not considering the correlation between the profits from titles. Table 5 is the correlations of the returns of the actions of the two problems.

In problem 1, all correlations are positive and less than 0.5, the correlations lowest are active x_1 with assets x_2 and x_3. As a result, the active x_1 has the highest proportion in the CAPM-driven portfolio. But that same asset has the highest proportion in the portfolio obtained by the LMM. In problem 2, positive and negative correlations imply that the CAPM will find the balance between these actions. The balance is realized with two correlations greater magnitude of different sign, it is between (ρ_{23}) and (ρ_{13}) correlations. As a result, the active x_3 is larger in selected portfolio by CAPM. One could say that in the problem 2, the magnitude and the sign of correlations differ over the assumption of the LMM, all positive and equal to one. But problem 2 calculated portfolio assets with the highest proportion is x_2, asset that has a high correlation (ρ_{23}), less than 0.5, and other low correlation (ρ_{12}). This portfolio is the highest performance at the lowest risk, if two portfolios obtained by CAPM and LMM are calculated with the LMM assumptions.

Table 4. Problem 2 investment portfolio risk and return.

Model		X_1	X_2	X_3	Sum	Diff
	Composition	0.1000	0.4000	0.5000	1	
CAPM	Return	0.9362	1.2249	1.05591	3.2170	0.6888
	Risk	--------	-------	-------	3.9058	
	Composition	0.1000	0.4000	0.5000	1	
CAPM risk by LMM	Return	0.9362	1.2249	1.05591	3.2170	3.7207
	Risk	2.1754	2.3454	2.4169	6.9377	
	Composition	0.0417	0.9576	0.0007	1	
LMM	Return	0.3904	2.9325	0.0015	3.3244	2.9630
	Risk	0.9072	5.3768	0.0034	6.2874	
LMM risk by CAPM	Composition	0.0417	0.9576	0.0007	1	2.1897

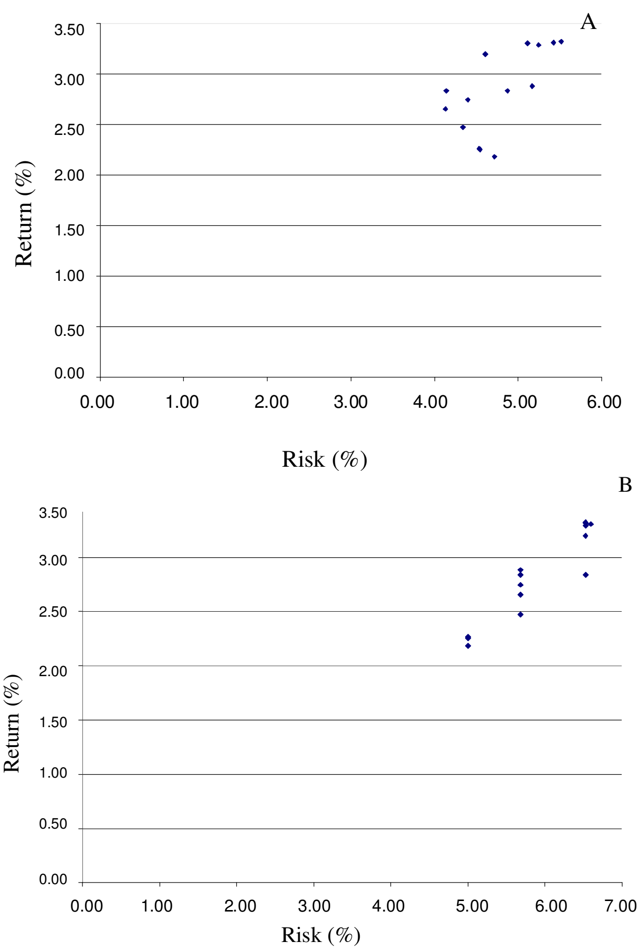

Figure 5. Points selected by the LMM 2 problem resolution process Space given by (A) CAPM and (B) LMM.

Conclusions

With the LMM, it is possible to solve two optimization problems simultaneously, maximize the performance and minimize the risk, and obtain an optimal investment portfolio.

Table 5. Correlation of the profit on assets problems 1 and 2.

Problem	ρ_{12}	ρ_{13}	ρ_{23}
1	0.128892614	0.106303068	0.42373978
2	-0.192360974	-0.344533899	0.42042912

In the BVM, no other composition of portfolios for a same scenario allows us to assess, from another point of view, the selection of a portfolio to take a competitive advantage. Finally, it is concluded that the assumptions of calculating of the risk will define the assets and their proportions that make up the investment portfolio.

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